List Pricing versus Dynamic Pricing: Impact on the Revenue Risk

Matthias Koenig
Lancaster University
Management School, UK
m.koenig@lancaster.ac.uk

Joern Meissner
Lancaster University
Management School, UK
joe@meiss.com

April 30, 2009

Abstract

We consider the problem of a firm selling multiple products that consume a single resource over a finite time period. The amount of the resource is exogenously fixed. We analyze the difference between a dynamic pricing policy and a list price capacity control policy. The dynamic pricing policy adjusts prices steadily resolving the underlying problem every time step, whereas the list pricing policy sets static prices once but controls the capacity by allowing or preventing product sales.

As steady price changes are often costly or unachievable in practice, we investigate the question of how much riskier it is to apply a list pricing policy rather than a dynamic pricing policy. We conduct several numerical experiments and compare expected revenue, standard deviation, and conditional-value-at-risk between the pricing policies. The differences between the policies show that list pricing can be a useful strategy when dynamic pricing is costly or impractical.

Keywords: revenue management, pricing, risk analysis, dynamic programming, capacity control

1 Introduction

Consider the following revenue management problem: the objective of a firm is the maximization of revenues from the production and sale of multiple products. A single resource is used for the production of the different products. A fixed capacity of this resource is available at decision time, and it has to be consumed within a given time horizon. The firm operates in a market which allows demand to be influenced by product pricing.

Pricing policies, which are asymptotically optimal for large growth demand and capacity, may show different behavior in the case of scarce capacity or demand. Maglaras and Meissner (2006) demonstrate that resolving the deterministic problem at each time step and implementing the resulting prices often offers better results than a static policy. Given the choice between dynamic and static pricing with capacity control, Gallego and van Ryzin (1994); Talluri and van Ryzin (2005) show that dynamic pricing should be preferred, as price flexibility allows for reducing simultaneously demand and increasing revenue.

However, changing the price can often involve significant costs and is not always feasible, as shown in Levy et al. (1998); Bergen et al. (2003) and Netessine (2006). Different types of costs accompanying price changes and overall costs can amount to one-fifth of a company's net margin, as the empirical investigation by Zbaracki et al. (2004) reveals. Continuous price changes might not be practical in settings, such as the example of supermarkets using price tags. From this perspective, a pricing policy without any price changes has its own benefits.

Without consideration to the costs of price changes, a dynamic pricing policy is likely to be more profitable than a static pricing. But when the costs exceed this benefit of dynamic pricing, static pricing becomes preferable. The cost of price changes might be less important when a long-term average effect is taken into account. But as this cost often involves fixed costs, they can significantly affect short-time objectives. Such circumstances can be given when fluctuations in working capital or revenue streams are unwanted. Then both expected revenue and downside risk must be balanced.

Thus, we need to investigate the question of how much price changes could cost a company that is still using a continuous price adaption strategy, instead of static pricing with the capacity for an allocation policy under uncertainty. In other words, we are interested in exploring what percent of expected revenue does a company risk using static pricing with control over capacity allocation. We approach this question by comparing results from numerical experiments conducted with these two different pricing policies. In particular, we use the dynamic pricing policy and the list price capacity control policy proposed by Maglaras and Meissner (2006). The dynamic pricing policy resolves the revenue management problem at each time step and updates the prices. This resolving policy is the most commonly used policy in practice. The list price capacity control policy is based on static prices and switches off the selling of a product after a distinct time period, depending on remaining capacity.

Using the same experimental settings, we compare the revenues that both policies achieve. The average revenues alone do not reveal the risk involved for a strategy. Therefore, in order to quantify risk we apply two additional measures on the results. Standard deviation serves as a measure of volatility and conditional-value-at-risk serves as risk measure. These measures help to gain more insight on when to prefer list pricing to dynamic pricing and vice versa.

The contributions of this paper are a comparison between one common policy for dynamic pricing and one for static pricing under risk consideration, with an emphasis on how the costs of price changes can affect the choice between policies.

The remainder of this paper is as follows: Section 2 reviews related literature. The problem layout is described in Section 3. Section 4 contains a brief description of the pricing policies evaluated in this paper. The used risk measures are explained in Section 5. Section 6 presents and discusses numerical results in terms of these measures. Finally, Section 7 concludes this paper.

2 Related Literature

The following are categories of related research: investigations of costs of price changes, analyses of dynamic pricing and static pricing, and risk considerations within dynamic pricing and revenue management.

The first category studies the cost of price changes by data analysis from a management perspective. Levy et al. (1997, 1998); Bergen et al. (2003) and Zbaracki et al. (2004) explore the cost of price changes in different industries. Levy et al. (1997) analyze the data of pricing systems of supermarket chains, derived from five different costs of price changes. They found that if the costs of price adjustment made the adjustments unprofitable they are not implemented. Levy et al. (1998) extend this analysis by describing the price adjustment process and involved decisions in more detail. Bergen et al. (2003) emphasize that implementing price changes is not costless and offer recommendations for a price change strategy at the managerial level. Zbaracki et al. (2004) report on the empirical data of costs for price adjustments. These empirical studies support the argument that the costs of price changes should be considered when applying dynamic pricing policies.

Considering the price changing costs, Netessine (2006) analyzes the dynamic pricing problem of a single product when only a limited number of price changes are allowed. He considers the timing of the price changes in a deterministic environment in cases of constraint and unconstraint capacity. This work focuses on an analytic solution for maximum expected revenue, whereas our paper looks at the question which price changing costs are acceptable under downside risk.

There are several papers which analyze dynamic and static pricing policies. Gallego and van Ryzin (1994, 1997) discuss the numerical results of static pricing in comparison to the optimal price. They conclude that for large sized problems with known demand functions and no constraint on price setting, there were no great benefits when using dynamic pricing. Cooper (2002) and Maglaras and Meissner (2006) present results of contrasting static and resolving pricing policies. Cooper (2002) provides an example showing that resolving policies does not necessarily lead to a better result, whereas the numerical results of Maglaras and Meissner (2006) show that its expected revenue is superior to static pricing in non-asymptotically settings. However, the policies generally assume a long-term perspective when studying expected revenue and a short-term view when risk sensitivity might be more important.

The last category comprises papers which deal with risk considerations in a revenue management setting. Feng and Xiao (1999) investigate risk in revenue management models. They introduce a risk factor in the form of sales variations into expected revenue. Lai and Ng (2005) present an idea based on stochastic programming for tackling risk in hotel revenue management. They include a risk trade-off factor between expected revenue and deviation into their stochastic network optimization model. This factor serves as a representation of risk aversion for decision makers. Levin et al. (2008) use a penalty term for the probability that revenues fall below a certain level. They suggest the use of their model for applications with a focus on a revenue target, such as event management.

Weatherford (2004) proposes an expected marginal seat utility (ESMU) employing utility theory to account for risk sensitivity in revenue management. Barz and Waldmann (2007) analyze an exponential utility function in capacity control models to incorporate risk-sensitivity. They employ a risk-sensitive Markov decision model in their approach.

Bertsimas and Thiele (2006) show robust and data-driven optimization that integrates uncertainty into decision making. Thiele (2006) presents a robust optimization approach to a single product pricing problem without knowing the demand function. Perakis and Roels (2007) analyze robust capacity controls for network revenue management using the maximin and minimax regret criteria.

Lim et al. (2008) extend their relative entropy approach from the single product, see Lim and Shanthikumar (2007), to multi products cases and a proposed a risk-sensitive pricing problem involving a risk sharing rule. Mitra and Wang (2005) discuss various risk indices for risk modeling of network revenue management problems.

Lancaster (2003) emphasizes the need for risk awareness in revenue management from the analysis of historical and ESMRb-simulated data. He advocates the introduction of risk measures based on value- at-risk for revenue management. Therefore, he derives a value-at-risk metric employing revenue per available seat mile (RASM). He proposed using a decision rule by a Sharpe ratio in order to find balancing risk and return strategies for airlines to deal with revenue volatility.

The papers which set up the last category examine specific important problems but do not provide a comparison of static and dynamic pricing under risk issues. This paper aims to produce such a comparison by measuring the risk inherent within these policies.

An overview of measures for risk analysis is given by Vose (2008) among others. Rockafellar (2007) gives a detailed discussion of the measures of deviation and of risk with regard to optimization. Luciano et al. (2003) argue that value-at-risk (VaR) is also a suitable risk measure for non-financial sectors. They use VaR to evaluate an inventory policy where the time horizon defines the period over a policy to be evaluated, and the confidence level specifies the worst cases to be considered regarding return.

3 Problem Description

3.1 Problem Setting

Our problem setting is described as a firm using a single resource for the production of multiple products. The number of capacity units of the single resource is denoted by C and the number of products by n. The products are indexed by i and each product i requires one unit of capacity. The resource must be used in a finite time horizon T, capacity cannot be replenished and the salvage value of remaining inventory at time T is assumed to be zero. The operating firm is assumed to be in a market with imperfect competition and can influence the demand for each product by varying its prices.

The prices at time $t \in [0, \ldots, T]$ are described by the n-dimensional vector $p(t)$. The corresponding demand is the n-dimensional vector $\lambda(p(t))$. The demand process is assumed to be a n-dimensional non-homogenous Poisson process. Further, the feasible sets are $\mathcal{P} \subseteq \mathbb{R}^n$ for the price vectors, $\mathcal{L} = \{x \geq 0 : x = \lambda(p), p \in \mathcal{P}\} \subseteq \mathbb{R}^n_+$ and for the achievable demand rate vectors so that $\lambda : \mathcal{P} \to \mathcal{L}$.

$\mathcal{L}$ is assumed to be a convex set, the demand function regular, and the existence of a continuous inverse demand function $p(\lambda)$ with $p : \mathcal{L} \to \mathcal{P}$. Thus, the expected revenue rate can be expressed as $R(\lambda) := \lambda' p(\lambda)$. We denote the transpose of a matrix x by x'. Time is discretized in small intervals of length δt and indexed by $t = 1, \ldots T$, so that $\mathcal{P}$(product i, arrival in $[0, \delta t]) = \lambda_i \delta t + o(\delta t)$ for all i and $o(x)/x \to 0$ as $x \to 0$.

In a slight abuse of notation, we write λ_i in place of $\lambda_i \delta t$ for the demand for product i. The random demand vector $\xi(t, \lambda)$ is Bernoulli with probabilities $\lambda(t) = \lambda(p(t))$.

3.2 Dynamic Pricing Problem

The dynamic pricing problem of Gallego and van Ryzin (1997) can be described then in the following form (e denotes the unit vector)

$$\max_{\{\lambda(t), t=1,\ldots,T\}} \left\{ \mathbb{E}\left[\sum_{t=1}^{T} p(\lambda(t))'\xi(t;\lambda)\right] : \sum_{t=1}^{T} e'\xi(t;\lambda) \leq C \ \text{ a.s.}, \lambda(t) \in \mathcal{L}\ \forall t \right\}. \tag{1}$$

Its associated Bellman equation for the expected revenue $V(x,t)$ starting at period t with x remaining units of capacity is

$$V(x,t) = \max_{\lambda \in \mathcal{L}} \left\{ \sum_{i=1}^{n} \lambda_i [p_i(\lambda) + V(x-1,t+1)] + (1 - e'\lambda)V(x,t+1) \right\}, \tag{2}$$

with boundary conditions

$$V(x,T+1) = 0; \forall x \text{ and } V(0,t) = 0; \forall t. \tag{3}$$

Equation 2 can be transformed to

$$V(x,t) = \max_{\lambda \in \mathcal{L}} \left\{ R(\lambda) - \sum_{i=1}^{n} \lambda_i \Delta V(x,t) \right\} + V(x,t+1), \tag{4}$$

where $\Delta V(x,t) = V(x,t+1) - V(x-1,t+1)$ denotes the marginal value of one unit of capacity as a function of the state (x,t). We define the aggregate rate of capacity consumption $\rho := \sum_{i=1}^{n} \lambda_i$. The set of achievable capacity consumption rates is then given by $\mathcal{R} := \{\rho : \sum_{i=1}^{n} \lambda_i = \rho, \lambda \in \mathcal{L}\}$. We denote

$$R^p(\rho) := \max_{\lambda} \left\{ R(\lambda) : \sum_{i=1}^{n} \lambda_i = \rho, \lambda \in \mathcal{L} \right\}, \tag{5}$$

as maximum achievable revenue rate under the constraint that the capacity consumption of all products is at rate ρ. Thus, Equation 4 can be rewritten as

$$V(x,t) = \max_{\lambda \in \mathcal{R}} \{R^p(\rho) - \rho \Delta V(x,t)\} + V(x,t+1). \tag{6}$$

3.3 Capacity Control Problem

The capacity control problem is related to the dynamic pricing problem as discussed by Maglaras and Meissner (2006). The price vector p is fixed now, and, thus, the demand rate vector $\lambda = \lambda(p)$ becomes fixed as well. In order to maximize expected revenue, the capacity allocation to products is optimized over time, as described by Lee and Hersh (1993). The products are assumed to be labelled such that $p_1 \geq p_2 \geq \ldots \geq p_n$. The firm controls if a product request i at time t is accepted which is modeled by the control $u_i(t)$. The dynamic capacity control problem is then

$$\max_{\{u(t),t=1,\ldots,T\}} \left\{ \mathbb{E}\left[\sum_{t=1}^{T} p'\xi(t;u\lambda)\right] : \sum_{t=1}^{T} e'\xi(t;u\lambda) \le C \text{ a.s.}, u_i(t) \in [0,1]\ \forall t \right\}, \tag{7}$$

where $u\lambda$ denotes the vector with coordinates $u_i\lambda_i$. The Bellman equation for the capacity control problem is

$$V(x,t) = \max_{u_i\in[0,1]} \left\{ \sum_{i=1}^{n} \lambda_i u_i [p_i + V(x-1,t+1)] + (1-u'\lambda)\, V(x,t+1) \right\}, \tag{8}$$

with the boundary conditions as given by Equation 3. Following the same steps as for the dynamic pricing problem, we can use the marginal value of capacity ΔV and write

$$\begin{aligned} V(x,t) &= \max_{u_i\in[0,1]} \left\{ \sum_{i=1}^{n} \lambda_i u_i p_i - u'\lambda \Delta V(x,t) \right\} + V(x,t+1) & (9)\\ &= \max_{\rho\in\mathcal{R}} \ \{R^c(\rho) - \rho\Delta V(x,t)\} + V(x,t+1), & (10) \end{aligned}$$

where $\rho = u'\lambda$ and the maximum revenue rate

$$R^c(\rho) = \max_{u} \{R(u\lambda) : u'\lambda = \rho, u \in [0,1]^n\},$$

when capacity consumption is at a rate equal to ρ.

Note that the optimal policy for dynamic pricing and the capacity control can be calculated using the formulation $\rho^*(x,t) = \operatorname{argmax}_{\rho\in\mathcal{R}} \ \{R(\rho) - \rho\Delta V(x,t)\}$, where $R(\cdot)$ is a concave increasing revenue function.

4 Policies

This section briefly describes the two pricing policies which are investigated in Section 6. Details about their structural properties can be found in Maglaras and Meissner (2006). We concentrate on the main aspects of the policies for their numerical implementation.

We assume that product i requests capacity consumption at rate of $a_i \ge 0$ units for each unit of demand and adapt $\rho = a'\lambda$ with $a = [a_1, \ldots, a_n]$. Then, static pricing with capacity constraints is described by the fluid formulation:

$$\max_{\{\rho(t), t\in[0,T]\}} \left\{ \int_0^T R^p(\rho(t))dt : \int_0^T \rho(t)dt \le C,\ \rho(t) \in \mathcal{R}\ \forall t \right\}. \tag{11}$$

The optimal solution of (11) is consumption of capacity at the constant rate

$$\bar{\rho}(t) := \min(\hat{\rho}, C/T) \quad \forall\ t, \tag{12}$$

with $\hat{\rho} = \text{argmax}_{\rho}\{R^p(\rho)\}$ (cf. Maglaras and Meissner, 2006). Note that C/T corresponds to the run-out rate that depletes capacity at time T. The corresponding demand vector is $\bar{\lambda} = \lambda^p(\bar{\rho})$ and its price vector $\bar{p} = p(\lambda^p(\bar{\rho}))$.

4.1 List-Price Capacity Control

The List-Price Capacity Control (LPCC) policy employs capacity control as explained in Section 3.3 for the solution of Equation 12. In order to determine the control u, two steps are implemented:

1. price with $\bar{p}$ and arrange products such that $\bar{p}_1/a_1 \ge \bar{p}_2/a_2 \ge \cdots \ge \bar{p}_n/a_n$,
2. calculate $\bar{\rho}(x,t)$ and control by u

$$\texttt{lpcc:} \quad \begin{cases} u_1(0,t) = 0 & \\ u_1(x,t) = 1 & \text{if } x > 0 \\ u_i(x,t) = \begin{cases} 1 & \text{if } \bar{\rho}(x,t) - \sum_{j<i} a_j \bar{\lambda}_j \ge 0 \\ 0 & \text{otherwise} \end{cases} & \text{for } i \ge 2. \end{cases} \tag{13}$$

The control excludes a product from further selling if the fluid solution for the future time period does not allow for the selling of the product.

4.2 Resolving

The dynamic pricing strategy is based on resolving the fluid formulation at discrete time points. This strategy can be expressed as:

$$\texttt{resolve:} \quad \lambda(x,t) = \lambda^r(\bar{\rho}(x,t)) \quad \text{and} \quad p(x,t) = p(\lambda(x,t)). \tag{14}$$

This policy is often used used in practice when readjustments can be done, e.g. at weekly intervals.

5 Risk Measures

The numerical assessment of the risk involved in the policies described in the previous section is evaluated by standard deviation as the measure for volatility and conditional-value-at-risk as the measure for risk.

5.1 Standard Deviation

Standard deviation measures dispersion of a random variable but is also often interpreted as risk measure. It provides information about how strongly the random variable varies from its expected value.

Consider a random variable X. The standard deviation is the square root of the moment-based measure variance σ^2 and it is defined as $\sigma(X) = \sqrt{E(X^2) - E(X)^2}$, where E denotes the expected value of a random variable.

Standard deviation is a volatility measure and considers results that are better and worse than expected value. Though risk is often assumed as the downside (worse than expected) return of an applied policy, measures other than standard deviation are required for evaluating only the downside distribution. Such measures comprise value-at-risk and conditional-value-at-risk.

5.2 Value-at-Risk and Conditional-Value-at-Risk

Value-at-risk (VaR) has become a popular risk measure in financial areas. Given a particular confidence level and time horizon, it measures the maximum expected loss on a portfolio of assets. The time horizon is chosen by the portfolio investor. Usual confidence levels are 95% or 99%. VaR lets the investor estimate the maximum loss of the portfolio in 95% or 99% of the cases. However, VaR can be a suitable risk measure for other industrial sectors as it is actually a percentile of a random variable that presents a distribution of returns.

Let the confidence level be denoted by $1 - \alpha$ and the random variable X represents earnings (here, lower $x \in X$ means greater loss). VaR can be then defined by the α-quantile of the random variable X with distribution function P and cumulative distribution function F_X as

$$VaR_\alpha(X) = \inf\{x : P(X \le x) \ge \alpha\} = \inf\{x : F_X(x) \ge \alpha\}.$$

The drawbacks of VaR are that it does not tell anything about the distribution of X below the VaR of the particular confidence level $1 - \alpha$ and it is not a coherent risk measure. For these reasons, conditional-value-at-risk (CVaR), also known as expected shortfall, is the preferred measure. We use CVaR in this paper due to the first reason given above, as the latter reason is not relevant for our evaluations.

CVaR does not have the deficiencies of VaR and can be interpreted as the expected value, given that the return is less or equal to the VaR value. Conditional-value-at-risk can be defined as:

$$CVaR_\alpha(X) = E(X | X \le VaR_\alpha(X)).$$

We follow the arguments by Luciano et al. (2003) and Lancaster (2003) for applications of VaR in other sectors than financial. As VaR can serve as the risk measure for evaluating policies, CVaR is useful for revenue management policies. It provides information about the downside of achieved revenue for a given confidence and a time horizon.

6 Experiments and Results

6.1 Experimental Setup

The two different policies are evaluated in several experiments. The demand was simulated using the same random numbers for each policy evaluation. The presented results for expected revenue, standard deviation and CVaR are actually averaged over 1,000 sample evaluations of a Monte Carlo simulation.

Instead of presenting the results of both investigated policies separately, our primary interest is in showing the differences between the results. The policy `resolve` is our benchmark. We expect that it performs better than the policy `lpcc` in terms of revenue as it is able to finer adapt its prices to demand. Therefore, we present the results as differences `resolve` minus `lpcc`. The differences are the benefit of using `resolve` when price changes are costless. This benefit also determines the costs that price adjustments might not exceed in order to prefer the policy `resolve` over `lpcc`.

We show the differences of achieved revenue and the differences of standard deviations as well as of CVaRs of both policies: a) $E(R_{\texttt{resolve}}) - E(R_{\texttt{lpcc}})$, b) $\sigma(R_{\texttt{resolve}}) - \sigma(R_{\texttt{lpcc}})$ and c) $CVaR_\alpha(R_{\texttt{resolve}}) - CVaR_\alpha(R_{\texttt{lpcc}})$, where the subscript on the revenue R denotes the policy. We use a value of $\alpha = 5\%$ for calculation of $CVaR_\alpha$ in our experiments. Again, it should be mentioned that the CVaR values are not losses but revenues for the worst cases, meaning that if the difference c) is greater than zero, `resolve` is less risky than `lpcc`.

We consider a linear demand function of the form $\lambda(p) = \Lambda - Bp$ in the following, where the vector $\Lambda = (\Lambda_i)$ denotes the market potential for product i and the matrix $B := (b_{ij})$ denotes the price and cross-price sensitivity for product i or between products i and j respectively.

The ordinates of the presented plots are in units of percent of the maximal expected revenue computed using (6, 10). The abscissas display the load factor $\hat{\rho} = T(\sum_{i=1}^{n} a_i \hat{\lambda})/C$ where $\hat{\lambda}$ is the result of (5) without the constraint. With the linear demand, it follows that $\hat{\lambda} = \text{argmax } \{\lambda' B^{-1}(\Lambda - \lambda) : \lambda \geq 0\} = (B^{-1} + B^{-1\prime})^{-1} \Lambda B^{-1}$. The load factor measures the relation between demand and available capacity. The figures show an interval range of the load factors from 0.5 (low demand, high capacity) to 4.0 (high demand, less capacity), which might be theoretical because the demand would be better estimated in the most realistic of scenarios.

6.2 Experiment without Cross-Price Elasticity Effects and Uniform Capacity Consumption

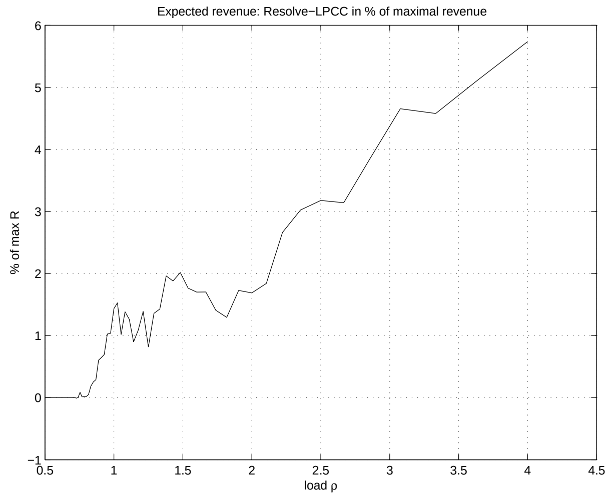
In order to illustrate the behavior of the different strategies, we first considered a simple numerical example using the same setup as done by Maglaras and Meissner (2006): two products consuming one unit of capacity per request, linear demand with $\Lambda = [0.3; 0.1]$, $B = [1\ 0; 0\ 6]$ and $T = 200$ time periods. The results are shown in Figure 1.

We observe that the advantage of `resolve` against `lpcc` decreases with decreasing load factor $\hat{\rho}$ (which corresponds to an increasing capacity C) from about 5.8% to 0% of maximal expected revenue. Further, standard deviation and CVaR demonstrate that `resolve` is significantly less risky when capacity is very scarce. In simulated cases with a load factor of $\hat{\rho} = 4$, it is about 28% of achievable revenue better than `lpcc` in the worst case of $\alpha = 5\%$. However, this benefit diminishes with increasing capacity and the difference of $\text{CVaR}_{5\%}$ favors neither of the strategies for $\hat{\rho} < 1.25$. The standard deviation of `lpcc` is actually lower than the standard deviation of `resolve` for a load factor $\hat{\rho} < 1.75$. Considering the results in asymptotical settings by Gallego and van Ryzin (1997); Maglaras and Meissner (2006), this seems coincident.

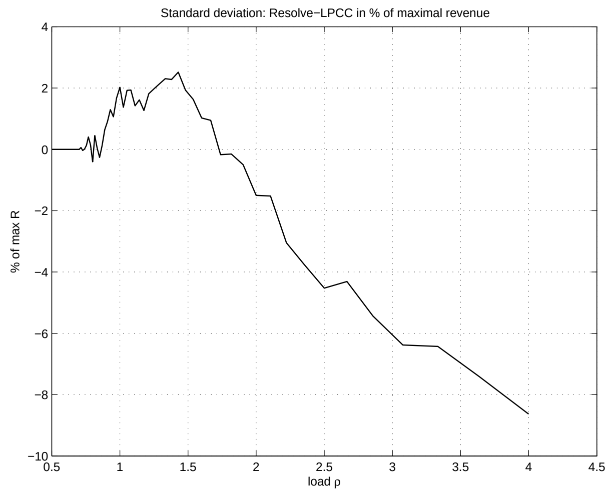
The most interesting behavior between both policies is in the range of the load factor between 0.75 and 1.75. Below 0.75 there is no clear difference and above 1.75 the outperformance of `resolve` against `lpcc` grows nearly linearly with increasing load factor. But between those values, the revenue achieved by `lpcc` is up to 2.2% less volatile, but on average also 2% less than the result of `resolve`. The CVaR advantage of `resolve` depreciates but is still up to 5% between $1.5 < \hat{\rho} < 1.75$. In those scenarios, list pricing offers a predictable revenue and risk estimation.

6.3 Experiment with Cross-Price Elasticity Effects and Uniform Capacity Consumption

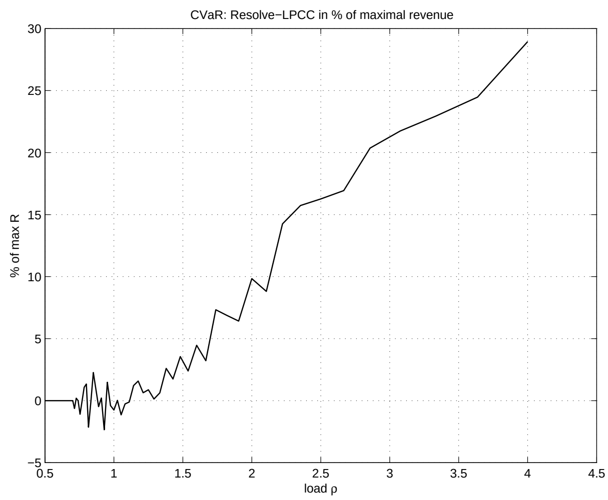
In order to investigate the cross-price elasticity effect, we use the same settings as before but change the matrix $B = [1\ -0.4; -0.6\ 6]$. The result of cross-price elasticity is shown in Figure 2.



(a) difference of revenues

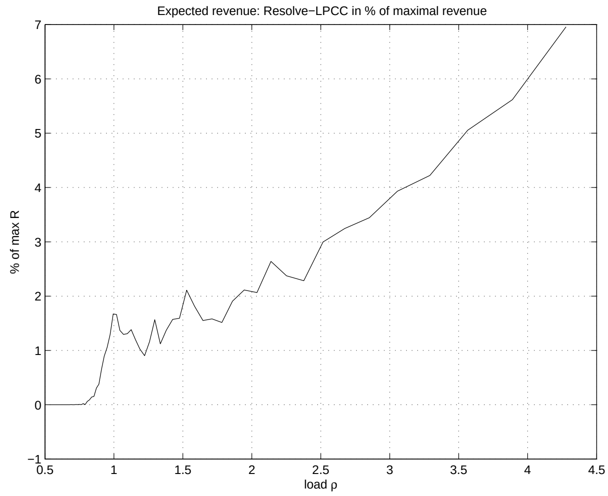


(b) difference of standard deviations

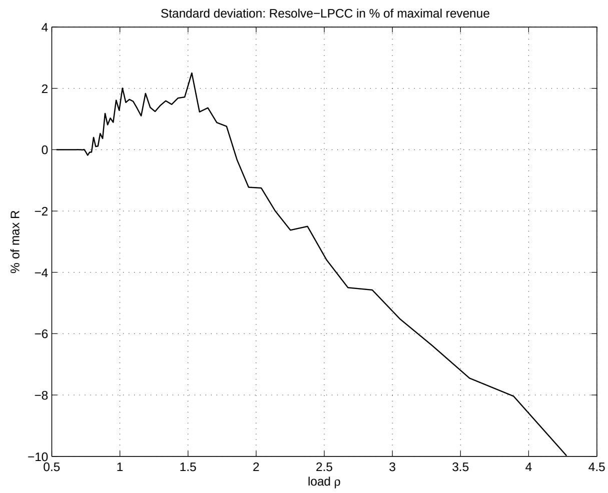


(c) difference of CVaRs ($\alpha = 5\%$)

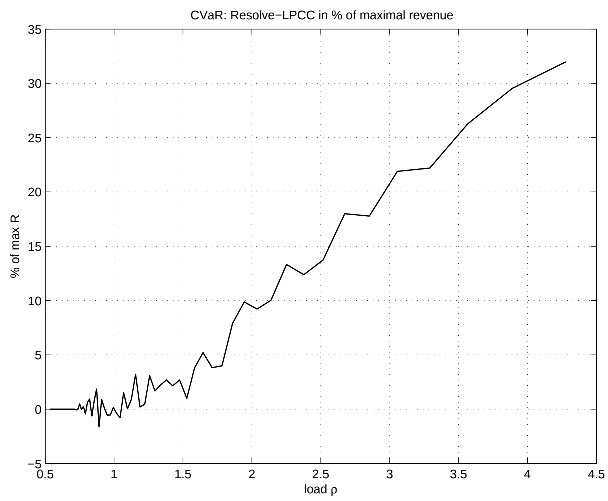
Figure 1: Results of Experiment 1: $B = [1\ 0; 0\ 6]$ and $a = [1\ 1]$.



(a) difference of expected revenues



(b) difference of standard deviations



(c) difference of CVaRs ($\alpha = 5\%$)

Figure 2: Results of Experiment 2: $B = [1\ \ -0.4; -0.6\ 6]$ and $a = [1\ 1]$

Comparing Figure 1 and Figure 2, no significant differences are observable between the examples with and the examples without cross-price sensitivity. A small distinction is identifiable in the plot of the expected revenue differences for load factors $1.0 \leq \hat{\rho} \leq 2.0$: the advantage of `resolve` is slightly better in the experiment with cross-price sensitivity, even though the conclusions of the experiment without cross-price sensitivity can be transferred to this experiment.

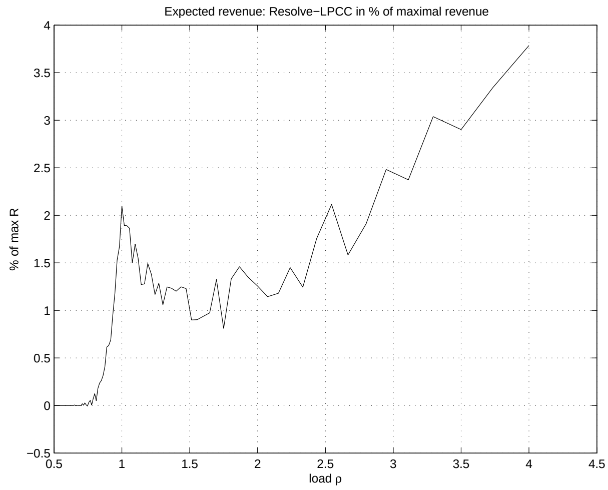
6.4 Experiments with Cross-Price Elasticity Effects and Non-Uniform Capacity Consumption

The next two examples use the same price/cross-price sensitivity matrix $B = [1\ -0.4; -0.6\ 6]$. Market potential and time period remain: $\Lambda = [0.3; 0.1]$, $T = 200$. The product consumption rates are $a = [1\ 2]$ for the experiment presented in Figure 3 and $a = [2\ 1]$ for the experiment in Figure 4.

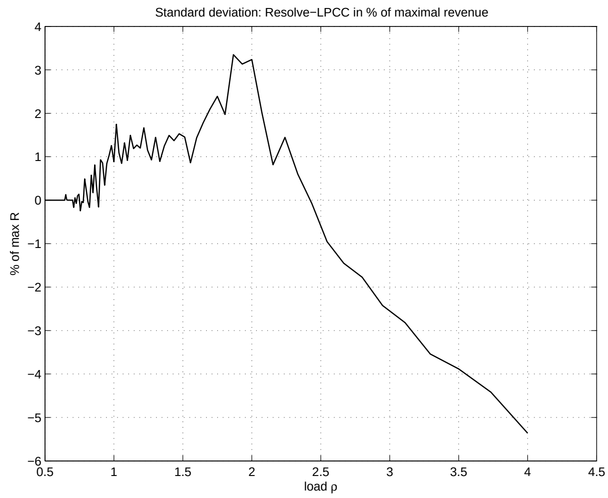
Generally, the results are similar to the uniform utilization of product consumption but contain some interesting differences in CVaR.

The plots of Figure 3 reveal similar behavior in the performance differences between both policies as in the previous results. Resolving the pricing problem at each time step has an advantage in expected revenue for a load factor $\hat{\rho} > 0.75$, a lower deviation for $\hat{\rho} > 2.5$ and less risk for $\hat{\rho} > 2.0$. The list pricing policy yields approximately 2% less revenue for load factor $\hat{\rho} \approx 1.0$ but performs little better up to a load factor $\hat{\rho} \leq 2.5$. On the other hand, its standard deviation is less than the resolving strategy for $\hat{\rho} \leq 2.5$. In this setting, however, combining all three plots of Figure 3, `lpcc` offers a reliability that is not worse than $\approx 2\%$ in revenue and risk when compared with `resolve` for a load factor $\hat{\rho} < 2.0$. For this, the reason is presented by the capacity consumption vector $a = [1\ 2]$; `resolve` cannot achieve much better than `lpcc` due to the lesser impact of the cheaper product. The benefit of operating only the higher valued product is less pronounced here.

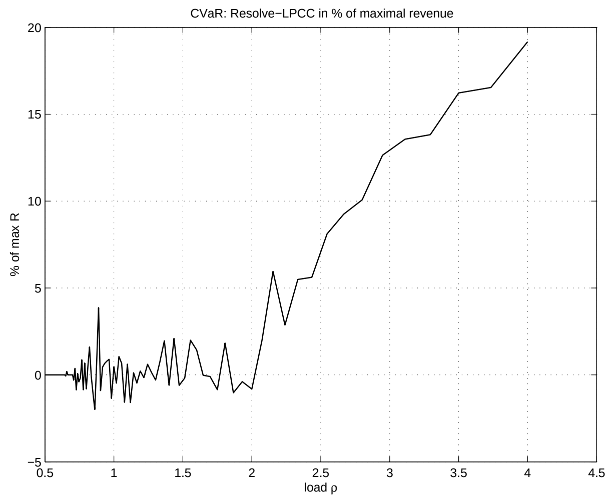
Figure 4 with $a = [2\ 1]$ illustrates this effect even more. The first product consumes more capacity, and the difference between price per unit of capacity consumption is less distinct than in the previous example. Not selling the first product can leave more remaining unused capacity. Thus, the results look similar to the previous results but the outperformance of `resolve` is substantial for increasing load factors. The advantage of `resolve`



(a) difference of expected revenues



(b) difference of standard deviations



(c) difference of CVaRs ($\alpha = 5\%$)

Figure 3: Results of Experiment 3: $B = [1 \ -0.4; -0.6 \ 6]$ and $a = [1 \ 2]$.

begins at a lower load factor ($\hat{\rho} > 1.5$) with a greater ascent as is the case with a uniform capacity consumption vector. The effect on the CVaR is also apparent. The difference in CVaR is highest for all examples, e.g. $\approx 7\%$ for $\hat{\rho} = 1.5$ whereas CVaR is $< 5\%$ here in the other examples.

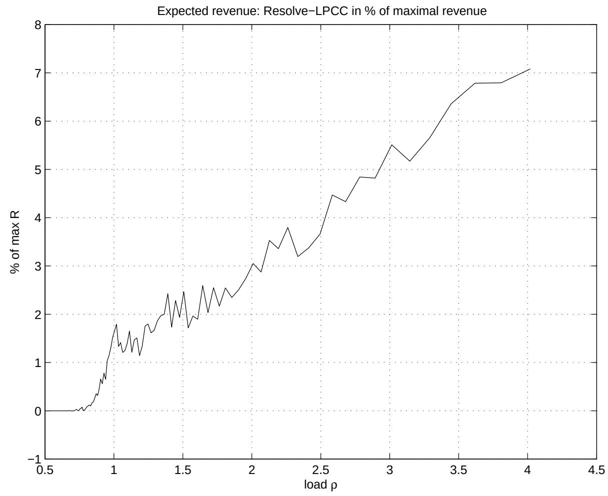
6.5 Discussion of Experiments

We observe that there can be significant differences between both policies, and for large load factors `resolve` clearly outperforms `lpcc`. However, realistic scenarios are for load factors $\hat{\rho} = 1.0 \pm 0.5$, and the result of comparison is less clear for deriving settings when the list pricing policy provides a benefit over the dynamic pricing policy and when price changes are costly. We assume that there are no costs associated with the list pricing policy, or that such costs are a base line for price adaption costs of the resolving pricing. If the costs for price changes are greater than the presented advantage of the resolving policy over list pricing with capacity control, then list pricing is the better policy.

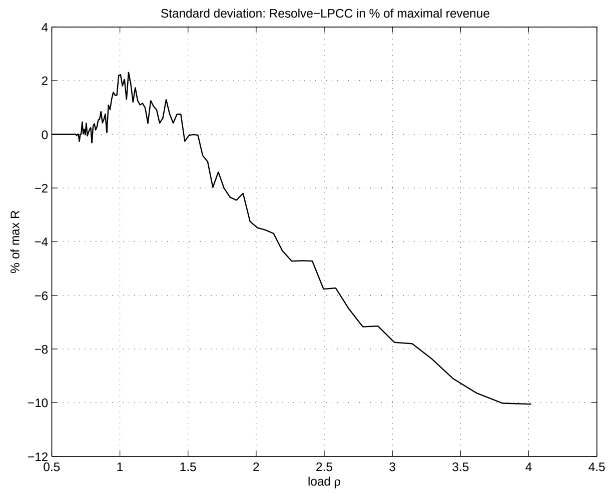
Considering only load factors $\hat{\rho} = 1.0 \pm 0.5$, the examples provide a lower border that the costs of price changes should not exceed. Otherwise, the resolving policy becomes more beneficial than the list pricing policy. For example, if the price changing costs more than 2.0% of maximum expected revenue, list pricing has an advantage in terms of expected revenue and volatility. The list pricing policy should be preferred in such settings as when the costs of price changes exceed this border and when risk considerations are not an issue.

This lower border is greater when risk aversion is important. It is up to 7% for load factor $\hat{\rho} = 1.0 \pm 0.5$ in the last of the examples. This is a substantial difference, as in realistic worst case scenarios, the resolving policy is better off at 7% than list pricing with capacity allocation control. However, this effect is caused by the product consumption vector which is known before and provides a basis for risk estimation.

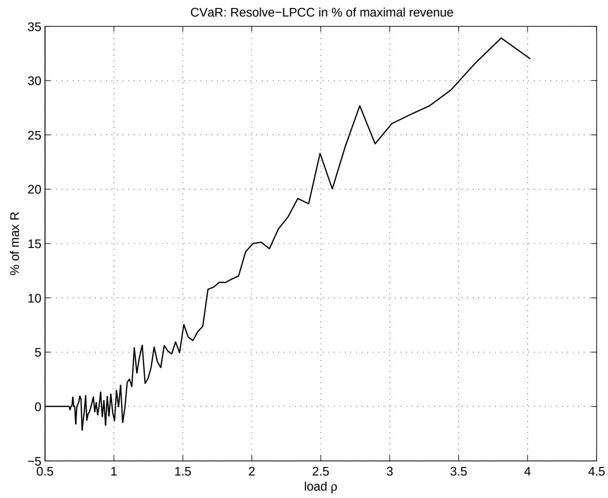
In general, for the case when capacity is scarce in relation to demand, the risk is higher for a list pricing than a resolving policy regardless whether standard deviation or CVaR is used as a measure. The difference declines when the available capacity increases and as a result, the load factor decreases Again, if knowledge about the operating load factor demand by capacity is given and there is certainty that it is only a few percentages greater



(a) difference of expected revenues



(b) difference of standard deviations



(c) difference of CVaRs ($\alpha = 5\%$)

Figure 4: Results of Experiment 4: $B = [1\ \ -0.4; -0.6\ 6]$ and $a = [2\ 1]$

than one, then list pricing can be the favorable policy to be taken into consideration for the costs of price changes.

7 Conclusions

In this paper, we have presented numerical results comparing a dynamic pricing policy, resolving the multi-product revenue management problem passing through time with a list pricing policy controlling the selling of products. We have been interested in which circumstances a list pricing policy might be favorable against a dynamic pricing policy when costs for price changes will be considered. Thus, we have described the results by looking at the differences between strategies regarding revenue, the volatility measure of standard deviation, and the risk measure of conditional-value-of-risk. These differences between dynamic and list pricing set the border that the costs of price changes must exceed so that a dynamic pricing strategy is an advantage.

We have found that in cases when there is substantial sufficient capacity in relation to demand (load factor between 0.5 and 1.5), the risk involved in the list pricing policy is only slightly higher than using the dynamic pricing policy. But the advantage of dynamic over list pricing policy in expected revenue, as well as in risk, increases significantly with decreasing capacity by holding demand constant.

If capacity utilization can be well-estimated and capacity consumption of the products is uniform, there might be a little disadvantage to choosing the list pricing policy. In our experiments, list pricing was worse off by ca. 2% of maximal expected revenue in revenue and risk downside risk measurement for load factor between 0.5 and 1.5. If there is confidence about working in such a setting, the cost of price changes of a dynamic pricing policy should be taken into account. These costs can be greater than the revenue disadvantage of choosing a list pricing policy. Therefore, the right choice of policy can be affected strongly by the costs as well as the applicability of implementing a dynamic pricing strategy.

Otherwise, if the costs of price adjustments are negligible or there is uncertainty of the setting, applying the dynamic pricing policy is the better choice as the list pricing policy might achieve less expected revenue and be more substantially risky in terms of maximal expected revenue (up to 7% in our experiments).

References

Barz, C. and K.-H. Waldmann. 2007. Risk-sensitive capacity control in revenue management, *Mathematical Methods of Operations Research*, 65(3), 565–579.

Bergen, M., M. Ritson, S. Dutta, D. Levy, and M. J. Zbaracki. 2003. Shattering the myth of costless price changes, *European Management Journal*, 21(6), 663–669.

Bertsimas, D. and A. Thiele. 2006. Robust and data-driven optimization: Modern decision-making under uncertainty, *Tutorials in Operations Research*, 95–122.

Cooper, W.L. 2002. Asymptotic behavior of an allocation policy for revenue management, *Operations Research*, 50(4), 720–727.

Feng, Y. and B. Xiao. 1999. Maximizing revenues of perishable assets with a risk factor, *Operations Research*, 47(2), 337–341.

Gallego, G. and G. van Ryzin. 1994. Optimal dynamic pricing of inventories with stochastic demand over finite horizons, *Management Science*, 40, 999–1020.

Gallego, G. and G. van Ryzin. 1997. A multiproduct dynamic pricing problem and its applications to network yield management, *Operations Research*, 45(1), 24–41.

Lai, K.-K. and W.-L. Ng. 2005. A stochastic approach to hotel revenue optimization, *Computers and Operations Research*, 32(5), 1059–1072.

Lancaster, J. 2003. The financial risk of airline revenue management, *Journal of Revenue and Pricing Management*, 2(2), 158–165.

Lee, T. and M. Hersh. 1993. A model for dynamic airline seat inventory controls with multiple seat bookings, *Transportation Science*, 27(3), 252–265.

Levin, Y., J. McGill, and M. Nediak. 2008. Risk in revenue management and dynamic pricing, *Operations Research*, 56(2), 326–343.

Levy, D., M. Bergen, S. Dutta, and R. Venable. 1997. The magnitude of menu costs: Direct evidence from large u. s. supermarket chains, *The Quarterly Journal of Economics*, 112(3), 791–825.

Levy, D., S. Dutta, M. Bergen, and R. Venable. 1998. Price adjustment at multiproduct retailers, *Managerial and Decision Economics*, 19, 81–120.

Lim, A.E.G. and J.G. Shanthikumar. 2007. Relative entropy, exponential utility, and robust dynamic pricing, *Operations Research*, 55(2), 198–214.

Lim, Andrew E. G., J. George Shanthikumar, and Thaisiri Watewal. 2008. Robust multi-product pricing, Tech. rep., University of Berkeley.

Luciano, E., L. Peccati, and D. M. Cifarelli. 2003. VaR as a risk measure for multiperiod static inventory models, *International Journal of Production Economics*, 81–82, 375–284.

Maglaras, C. and J. Meissner. 2006. Dynamic pricing strategies for mulitproduct revenue management problems, *Manufacturing & Service Operations Management*, 8(2), 136–148.

Mitra, D. and Q. Wang. 2005. Stochastic traffic engineering for demand uncertainty and risk-aware network revenue management, *IEEE/ACM Transactions on Networking*, 13(2), 221–233.

Netessine, S. 2006. Dynamic pricing of inventory/capacity with infrequent price changes, *European Journal of Operational Research*, 174(1), 553–580.

Perakis, G. and G. Roels. 2007. Robust controls for network revenue management, Tech. rep., Decisions, Operations, and Technology Management, University of California.

Rockafellar, R.T. 2007. Coherent approaches to risk in optimization coherent approaches to risk in optimization under uncertainty, *Tutorials in Operations Research*, 38–61.

Talluri, K.T. and G.J. van Ryzin. 2005. *The Theory and Practice of Revenue Management*, Springer.

Thiele, Aurelie. 2006. Single-product pricing via robust optimization, Tech. rep., Lehigh University.

Vose, D. 2008. *Risk Analysis: A Quantitative Guide*, Wiley, 3rd edn.

Weatherford, L.R. 2004. EMSR versus EMSU: Revenue or utility?, *Journal of Revenue and Pricing Management*, 3(3), 277–284.

Zbaracki, M. J., M. Ritson, D. Levy, S. Dutta, and M. Bergen. 2004. Managerial and customer costs of price adjustment: Direct evidence from industiral markets, *The Review of Economics and Statistics*, 86(2), 514–533.