COLUMBIA BUSINESS SCHOOL

Lecture 12

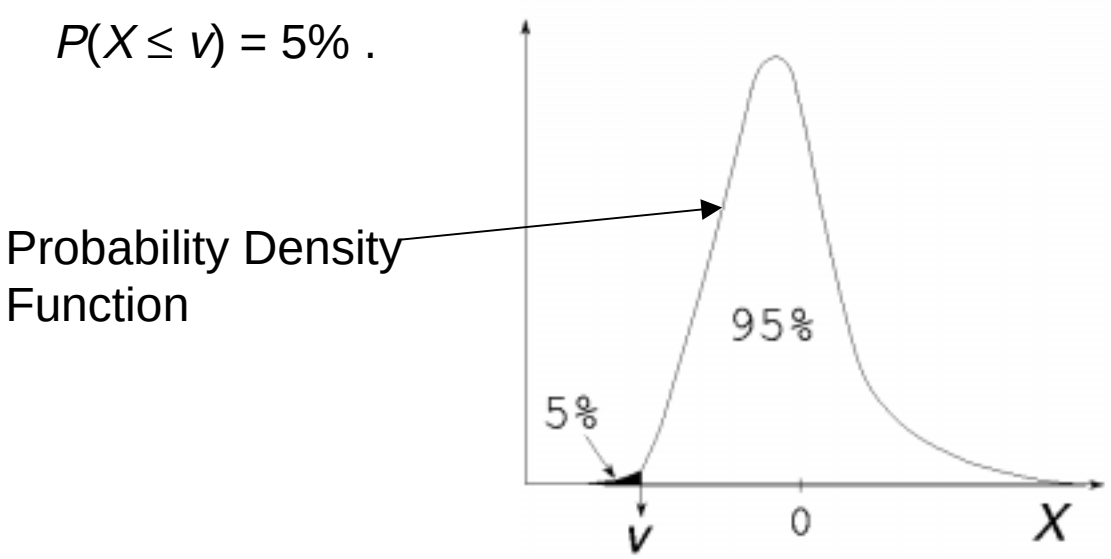
- Introduction to Value at Risk (VaR)
  - Computing VaR for a Single Bond
  - Computing VaR for a Bond Portfolio
  - Risk Management Systems

- Course Wrap Up

Value at Risk (VaR): Definition and Background

- VaR is a method for assessing financial risk. It has become a standard tool for many financial firms, asset managers, and non-financial firms. Regulators worldwide are moving to encourage or enforce the use of VaR or similar risk measures.
- Let X represent the daily profit or loss of a firm. Value at Risk (VaR) at a 95% confidence level is the number v satisfying

$$P(X \leq v) = 5\% \ .$$



- It is most typical to report Value at Risk as $-v$. For example, if v = -\$30 million, a bank would say that its daily VaR at a 95% confidence level is \$30 million. That is, there is only one chance in 20 that a one-day loss of \$30 million or more will occur.

Industry Use of VaR

- Huge losses have been suffered by Barings, Metallgesellschaft, Orange County, Daiwa, Askin Capital Management, and others. Many firms and investors want more information about financial risk.
- Every morning at Chase the senior VP in charge of global risk management receives a 30-page report summarizing the Value at Risk (VaR) of the bank.
- J.P. Morgan handles over 20,000 transactions per day with volume exceeding $50 billion. Every day their global financial risk is analyzed and summarized in a 4:15 p.m. VaR report which is sent to the chairman of the board.
- Mobil uses daily VaR to measure its exposure to interest rates, FX rates, and oil and gas prices.
- The Chrysler pension fund uses the RAROC 2020 risk-management system developed by Bankers Trust to compute Value at Risk.
- Merck developed its own risk-management system using a VaR-like approach (see previous lecture and readings).

Support for VaR by Regulators and Industry Groups

- The Basel Committee on banking supervision sets minimum-capital requirements for commercial banks in the European Union. In April 1995 they mandated capital requirements based on VaR computations to take effect by the end of 1997.
- In December 1995 the U.S. Securities and Exchange Commission issued a proposal that would require companies to disclose derivatives activities in one of three formats, with VaR as one alternative.
- A major recommendation of the 1993 Group of Thirty (G-30) report is to assess financial risk using a VaR system.
- Proposals recommending the use of VaR have been issued by Moody's and Standard and Poor's (rating agencies) and by the trade group ISDA (International Swap and Derivatives Association).

A good reference is *Value at Risk: The New Benchmark for Controlling Derivatives Risk*, by Philippe Jorion, 1997, Irwin (which is on reserve in the Business school library).

Value at Risk for a Single Bond

- Today: The Putnam government-bond fund owns $100-million face amount of a 30-year zero-coupon bond. The current bond yield is 8.00% and its price is $9.94 (per $100 face amount). The total market value is $9.94 million.
- Tomorrow: The change in the bond yield is unknown, but is believed to be normally distributed with a mean of 0.0% and a standard deviation of 0.10%.
- *What is Putnam's daily Value at Risk at a 95% confidence level?*

- Bond-pricing formula:

$$P = \frac{100}{(1+y)^t},$$

where P is the price of the bond, 100 is the face value of the bond, y is the yield to maturity, and t is the number of years to maturity.

Today: For y = 8% and t = 30 years,

$$P = \frac{100}{(1+0.08)^{30}} = 9.94.$$

Computing Putnam's Daily P&L

- Suppose the bond's yield increases by 0.09%, i.e., its yield tomorrow is 8.09%. What is Putnam's P&L for the day?
- If the yield tomorrow is y' = 8.09%, then tomorrow's price is

$$P' = \frac{100}{(1+0.0809)^{30}} = 9.69.$$

The loss is $P' - P$ = 9.69 - 9.94 = \$0.25 per \$100 face amount. So the total loss for the day is

$$\$0.25 \text{ million},$$

i.e., \$250,000.

- To compute VaR, we need to simulate the bond yield tomorrow. The specified model is

$$y' = y + \Delta y,$$

where y = 8.0% and

$$\Delta y \sim N(\mu = 0.0\%, \sigma = 0.10\%).$$

Putnam's daily P&L is given by

$$P' - P = \frac{100}{(1+y')^{30}} - 9.94. \quad (\text{in \$million})$$

A Model for the 30-Year Bond Yield Tomorrow

- The model for the 30-year bond yield tomorrow is

$$y' = y + \Delta y,$$

where $y = 8.0\%$ and

$$\Delta y \sim N(\mu = 0.0\%, \sigma = 0.10\%).$$

- This model can be estimated using historical data. Here we use 50 days worth of data:

Day	Yield (in %)	Yield Change (in %)
1	7.59	NA
2	7.73	0.14
3	7.70	-0.03
4	7.79	0.09
…	…	…
49	8.08	0.03
50	8.00	-0.08

- The standard deviation of yield changes is 0.10%, which can be computed in a spreadsheet using the =STDEV function.

Spreadsheet for Putnam's Daily P&L Simulation

	A	B	C	D	E	F	G	H
1	PUTNAM.XLS		**Putnam's Daily VaR Simulation**					
2								
3	Today	8.00%	Yield		Std dev of yield change			0.10%
4		30	Maturity (in yrs)		Yield change			**0.09%**
5		9.938	Price					
6								
7	Tomorrow	8.09%	Yield					
8		9.692	Price					
9	Putnam's							
10	P&L	**-0.245**	(in $million)					

Assumption cell H4: Normal with mean = 0, std dev = H3

Forecast cell B10 named P&L

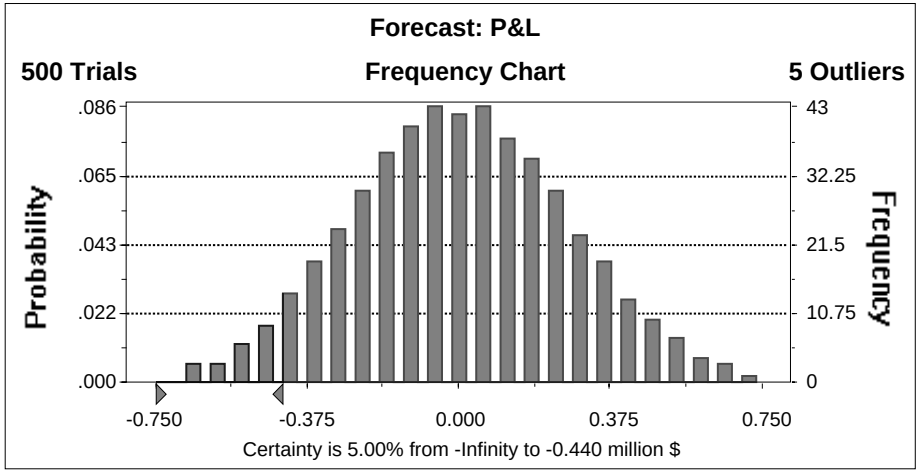
- Spreadsheet formulas:
  Cell B5: =100/(1 + B3)^B4, i.e., $P = 100/(1+y)^{30}$
  Cell B7: =B3 + H4, i.e., $y' = y + \Delta y$
  Cell B8: =100/(1 + B7)^B4, i.e., $P' = 100/(1+y)^{30}$
  Cell B10: =B8 - B5, i.e., P&L = $P' - P$ (in $million)
- In the Crystal Ball "Run Preferences," set the maximum number of trials to 500, the random number seed to 123, and for the sampling method choose "Latin Hypercube." This sampling method requires somewhat more computer memory, but gives more accurate results. The spreadsheet is now ready to run a Crystal Ball simulation.

Putnam Simulation Results

- To find Putnam's daily VaR at a 95% confidence level, we need to find v such that

$$P(X \leq v) = 5\% \ ,$$

where X is the random variable representing Putnam's daily P&L. After the Crystal Ball simulation is run, view the P&L forecast window:



- In the "P&L" forecast window, move the right arrow until the value in the certainty window reads 5%. This happens at a value of -0.440 million. That is, Putnam's daily VaR is $440,000.

Value at Risk for a Portfolio of Bonds

- Today: The Premier government bond fund owns three bonds with cash flows shown in the table (all cash flows in $ million):

	Year 1	Year 5	Year 10	Year 30
Bond 1	104	0	0	0
Bond 2	20	60	400	0
Bond 3	25	25	25	400

- The current yields by maturity (often called the *yield curve*) are given in the next table:

	Year 1	Year 5	Year 10	Year 30
Yield	4.00%	5.00%	6.50%	8.0%

- The current market value of bond 3 is:

$$\frac{25}{1.04^1}+\frac{25}{1.05^5}+\frac{25}{1.065^{10}}+\frac{400}{1.08^{30}}=\$96.70\text{ million}$$

Similarly, the market value of bond1 is $100, bond2 is $279.33, and the total value of the Premier bond fund is $476.03 (million).

- *What is Premier's daily Value at Risk at a 95% confidence level?*

A Model for the Yield Curve Tomorrow

- For the 1-year, 5-year, 10-year, and 30-year yields, we can use the models

$$y'_i = y_i + \Delta y_i, \text{ for } i = 1, 5, 10, 30$$

where y_i is given by the current yield curve and

$$\Delta y_i \sim N(\mu = 0.0\%, \sigma_i).$$

with $\sigma_1 = 0.10\%$, $\sigma_5 = 0.13\%$, $\sigma_{10} = 0.12\%$, and $\sigma_{30} = 0.10\%$. These models can be estimated using historical data of yield *changes* (all data in %):

Day	1-Year Yld Chng	5-Year Yld Chng	10-Year Yld Chng	30-Year Yld Chng
1	NA	NA	NA	NA
2	0.09	0.12	0.11	0.14
3	- 0.01	- 0.02	- 0.02	- 0.03
4	0.11	0.08	0.07	0.09
…	…	…	…	…
49	0.02	0.03	0.03	0.03
50	- 0.09	- 0.10	- 0.07	- 0.08
Std dev	0.10	0.13	0.12	0.10

The standard deviations can be computed in a spreadsheet using the =STDEV function.

Correlation of Yield Curve Changes

- When we have multiple sources of uncertainty, in addition to specifying each model individually, we need to specify the *correlations* between models. Using the same historical data, we can estimate a correlation matrix of yield changes:

	1-Year	5-Year	10-Year	30-Year
1-Year	1.0			
5-Year	0.9	1.0		
10-Year	0.8	0.9	1.0	
30-Year	0.7	0.8	0.9	1.0

- The entries above the diagonal are given by symmetry, e.g.,

  Correl(Δy_5, Δy_{30}) = Correl (Δy_{30}, Δy_5) = 0.8

- The entire correlation matrix can be estimated in Excel using the correlation tool. This tool can be accessed from the main menu by choosing “Tools” then “Data Analysis” and then “Correlation.”

Spreadsheet for Premier's Daily P&L Simulation

	A	B	C	D	E	F	G	H	I
1	PREMIER.XLS		**Premier's Daily VaR Simulation**						
2		*Today*	1	5	10	30	Year		
3			4.00%	5.00%	6.50%	8.00%	Yield		
4		Value	0.9615	0.7835	0.5327	0.0994	Discount factor		
5	Bond 1	100.00	104	0	0	0			
6	Bond 2	279.33	20	60	400	0			
7	Bond 3	96.70	25	25	25	400			
8	Total	476.03							
9			0.10%	0.13%	0.12%	0.10%	Std dev of yield change		
10		*Tomorrow*	**-0.07%**	**-0.01%**	**-0.04%**	**-0.02%**	Yield change		
11			3.93%	4.99%	6.46%	7.98%	Yield		
12		Value	0.9622	0.7839	0.5347	0.0999	Discount factor		
13	Bond 1	100.07	104	0	0	0			
14	Bond 2	280.17	20	60	400	0			
15	Bond 3	96.99	25	25	25	400			
16	Total	477.23							
17	P&L	**1.20**	(in $million)						

Assumption cells C10:F10, each Normal with mean =0, and std dev from row 9

Forecast cell B17 named P&L

- Key Spreadsheet formulas:

  Cell C4: =1/(1+C3)^C2 (copied to D4:F4)

  Cell B5: =SUMPRODUCT($C$4:$F$4,C5:F5) (copied to B6 and B7)

  Cell C8: =SUM(B5:B7)

  Cell C11: =C3 + C10 (copied to D11:F11)

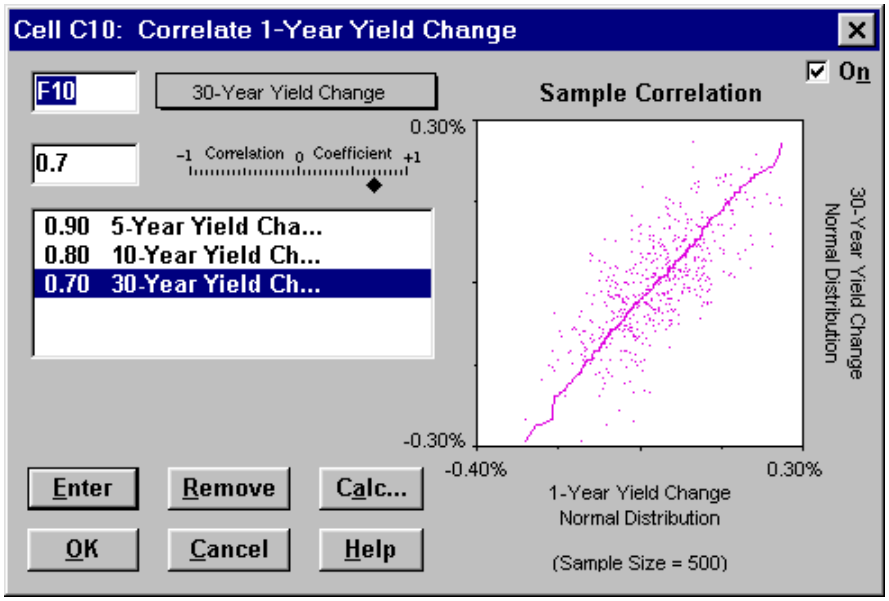
  Cell C12: =1/(1+C11)^C2 (copied to D12:F12)

  Cell B13: =SUMPRODUCT($C$12:$F$12,C13:F13) (copied to B14 and B15)

  Cell B17: =B16 - B8, i.e., P&L in $million

Specifying the Correlations in Crystal Ball

- First define the assumption cells C10:F10 as usual. Then return to the C10 assumption cell window and click on "Correlate."



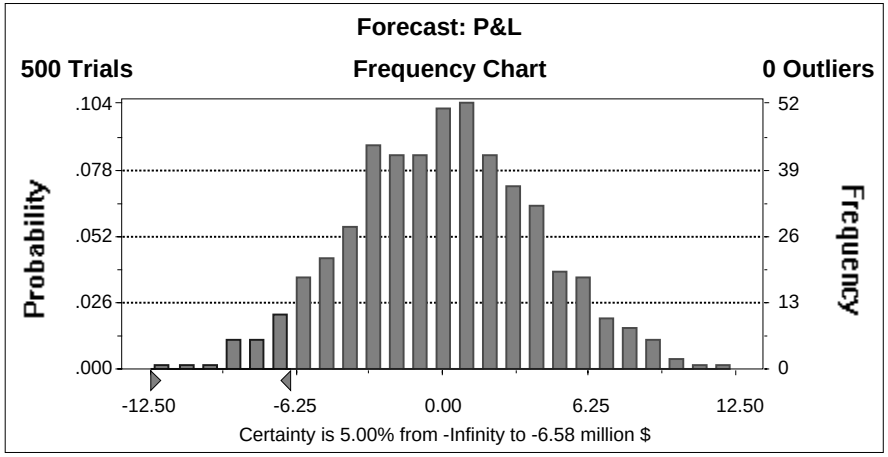
- Type =D10 in the "<Select Assumption>" window and type 0.9 in the correlation window. Then click on "Enter." Then type =E10 and 0.8 in the same windows and click on "Enter." Then type =F10 and 0.7 in the same windows and click on "Enter" and the window should look as above. Click on "OK." Similarly, add correlations to cells D10 and E10.

Premier Simulation Results

- To find Premier's daily VaR at a 95% confidence level, we need to find v such that

$$P(X \leq v) = 5\% ,$$

where X is the random variable representing Premier's daily P&L. After running the Crystal Ball simulation with the usual parameters (500 trials, seed 123, and Latin hypercube sampling), view the P&L forecast window:



- In the "P&L" forecast window, move the right arrow until the value in the certainty window reads 5%. Premier's daily VaR is $6.5 million.

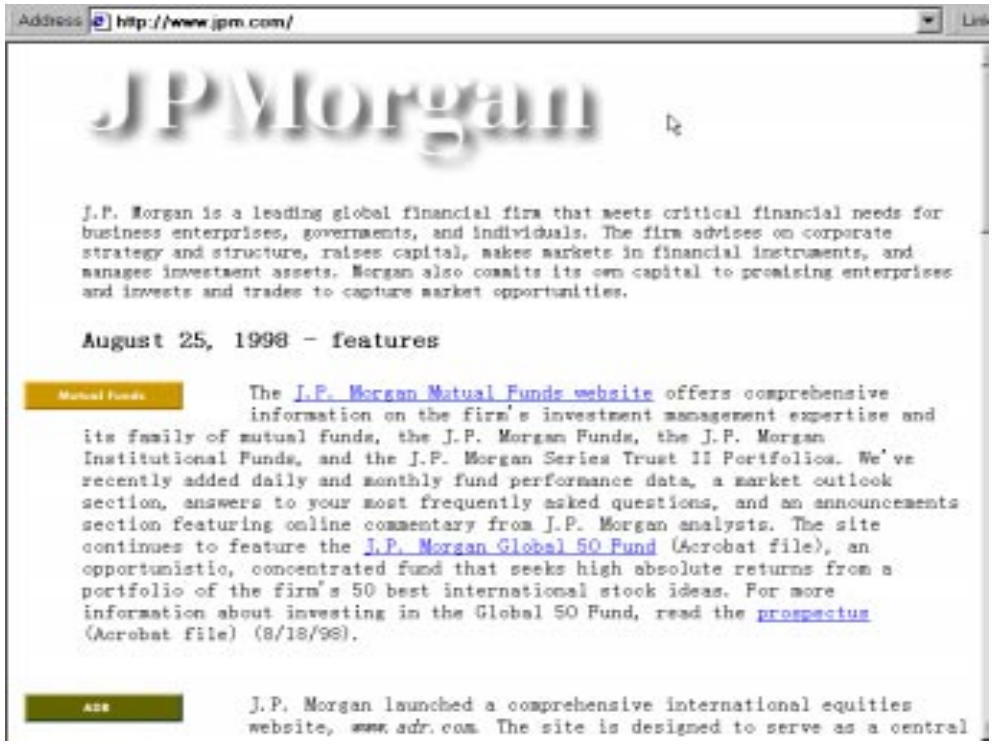
Comparison of VaR Results

	Putnam	Premier
Market Value (in $million)	9.94	476.0
Daily VaR (in $million)	0.44	6.5
Daily VaR	4.4%	1.4%

- Although Putnam's VaR is smaller on an absolute basis, it is far larger on a relative basis.
- A recent NYU survey found that 60% of pension funds use Value at Risk to measure market risk.
- Commercial banks in the European Union will have a capital charge based on a two-week VaR computation at a 99% confidence level.
- Running a VaR system on a daily basis requires access to high-quality data. To fill this need and promote itself as a premier risk-management advisor, in October 1994 J.P. Morgan released its RiskMetrics™ system publicly on the Internet.
- Many competing systems are now being sold commercially (e.g., Banker's Trust RAROC 2020 risk management system, Chase Manhattan's Charisma, C·ATS Software CARMA system).

J.P. Morgan's RiskMetrics™

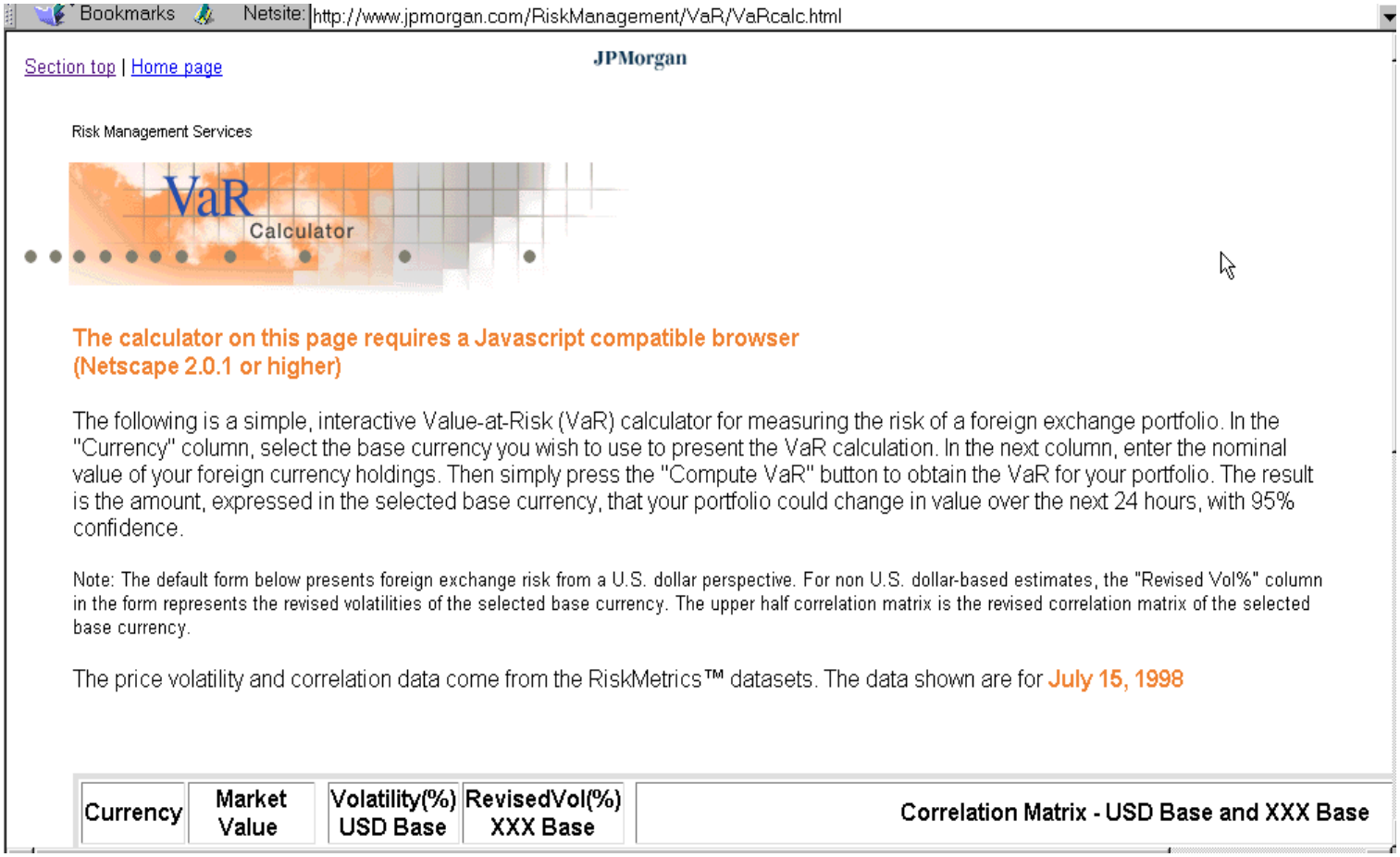
- J.P. Morgan's homepage (http://www.jpmorgan.com):



From here you can follow links to download RiskMetrics™ data, run an interactive VaR calculator, or download its FourFifteen VaR software for use with RiskMetrics™.

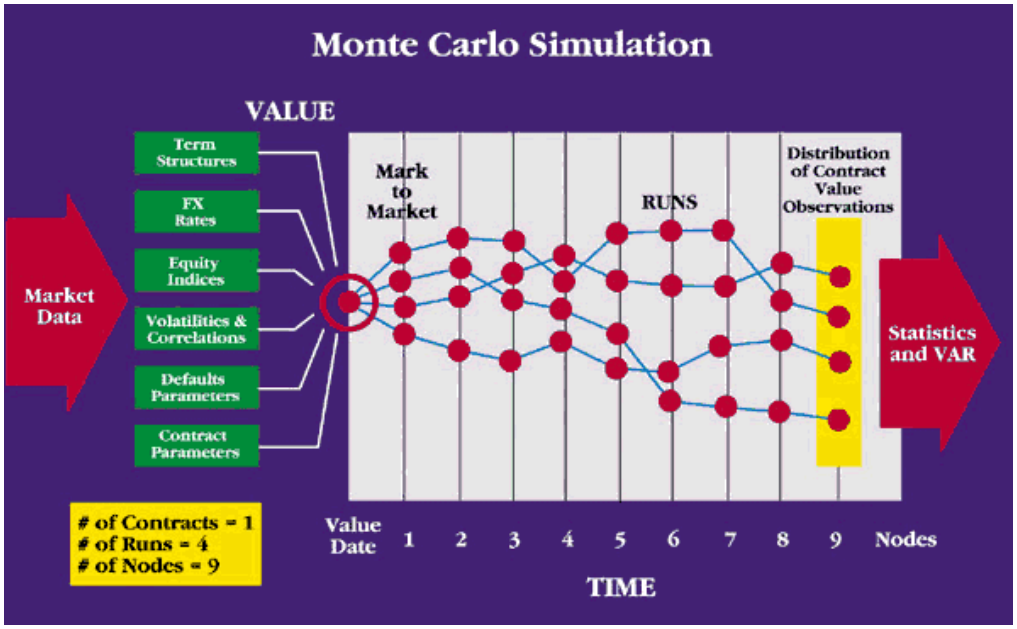
J.P. Morgan's RiskMetrics™ (continued)

- RiskMetrics™ provides over 300,000 volatility and correlation numbers daily which cover
  - bonds, money markets, FX, and equity indices
  - 15 countries/currencies



C·ATS Software, Inc.
CARMA Risk Management Package

- The homepage for C · ATS Software, Inc. is http://www.cats.com. Follow the links to “Our products” and then “Risk Management Solutions” to get to information about their CARMA package. Here is a screen from their website:



Also see the Banker’s Trust RAROC 2020 risk management system (http://www.bankerstrust.com).