Competition under Capacitated Dynamic Lot Sizing with Capacity Acquisition

Hongyan Li[1] and Joern Meissner[2]

[1] *CORAL, Department of Business Studies, Aarhus School of Business, Aarhus University, Denmark.*
hojl@asb.dk
[2] *Department of Management Science, Lancaster University Management School, United Kingdom.*
joe@meiss.com

August 20, 2010

Abstract

Lot-sizing and capacity planning are important supply chain decisions, and competition and cooperation affect the performance of these decisions. In this paper, we look into the dynamic lot sizing and resource competition problem of an industry consisting of multiple firms. A capacity competition model combining the complexity of time-varying demand with cost functions and economies of scale arising from dynamic lot-sizing costs is developed. Each firm can replenish inventory at the beginning of each period in a finite planning horizon. Fixed as well as variable production costs incur for each production setup, along with inventory carrying costs. The individual production lots of each firm are limited by a constant capacity restriction, which is purchased up front for the planning horizon. The capacity can be purchased from a spot market, and the capacity acquisition cost fluctuates with the total capacity demand of all the competing firms. We solve the competition model and establish the existence of a capacity equilibrium over the firms and the associated optimal dynamic lot-sizing plan for each firm under mild conditions.

Keywords: Game theory, capacity optimization, competition, lot sizing, approximation, equilibrium

1 Introduction

One of the fundamental problems in operations management is determining the investment in capacity. A firm's capacity determines its maximal potential production. To acquire capacity is usually cost and time consuming, and once the investment is made, the cost is often partially or completely irreversible, as installed capacity is difficult to adjust in the short term. Moreover, the decision on how much capacity to acquire also strongly influences the action space for future operations planning. To invest in too much capacity wastes resources that could be used for other important operation activities, such as new product development and marketing; to invest in too little capacity means long waiting times, missed sales opportunities and lost revenue. Therefore, it is necessary to find an effective and comprehensive method to determine the proper capacity configuration for operations.

Increasing the capacity does not necessarily improve the operational performance, even if the product profit margins are large, because capacity acquisition cost is usually negative correlated to the production cost and often affected by the competitive resource environment. In addition, the competitors' other decisions, such as the timing of production and quantity, also affect capacity acquisition cost and investment performance. Game theoretic modelling has been an effective method of describing and solving competition problems. In this paper, we solve a game-theoretic model of capacity competition problem over a finite-period planning horizon for a multiple-firm industry that uses a common resource to produce its products. For each firm, its best-response problem is a single-item capacity acquisition and lot-sizing problem.

The best-response problem considers a single-production facility that produces a single product item to satisfy a deterministic demand stream. The best-response problem for individual firms simultaneously determines an optimal capacity and a lot size plan over the planning horizon. The capacity acquisition, production and inventory holding costs are considered. We formulate the problem as a cost minimizing Mixed Integer Non-Linear Programming (MINLP) model. This general problem class is impossible to solve using a polynomial time algorithm. Thus, we discretize the possible capacity choices and solve it for each of those. The major difference between the best response problem and the classical capacitated lot-sizing problems is that the capacity level is an internal decision in our model.

Given the capacity competition model, we discuss the capacity equilibrium and associated optimal dynamic lot-sizing plans by analyzing the resulted best-response problem. We introduce an approximation for a firm's best response function, showing through a numerical study that its use results in only a minor difference to the actual cost figures but still has desirable properties. We then proceed to analyze the competitive problem and show the existence of an equilibrium under modest assumptions. To the best of our knowledge, this is the first study to address lot-sizing problems considering resource competition. Moreover, since the complexity of the capacity competition problem, the approximated solutions are acceptable in practice.

The remainder of this paper is organized as follows. We review the relevant studies in Section 2. Section 3 introduces the relevant notation and the basic competitive model. Section 4 first describes the best-response problem that an individual firm faces when making its purchasing and lot-sizing decisions. In Section 5, we show our suggested solution in a structure of the game which results in an equilibrium following a standard procedure. Finally, a computational study and numerical examples are discussed in Section 6.

2 Literature Review

The aim of capacity–acquisition decisions is to select the proper capacity that not only satisfies demand completely, but also minimizes the total capacity acquisition and lot-sizing cost. The research on capacity investment problems includes two main streams, the traditional mathematical programming models and the economic models.

Traditional mathematical programming methods have been applied to capacity-acquisition problems ever since research efforts first took notice of them. The flexible capacity investment and management problems arose and were addressed at a relatively early stage. Fine and Freund (1990) present a two-stage stochastic programming model and an analysis of the cost-flexibility trade-offs involved in the investment in product-flexible manufacturing capacity for a firm. They address the sensitivity of the firm's optimal capacity investment decision to the costs of capacity, demand distribution and risk level. Also, van Mieghem (1998) studies the optimal investment problem of flexible manufacturing capacity as a function of product prices, investment costs and demand uncertainty for a two-product production environment. He suggests finding the optimal capacity by solving a multi-dimensional news-vendor problem assuming continuous demand and capacity. Netessine et al. (2002) propose a one-period flexible-service capacity optimization and allocation model taking the capacity acquisition, usage, and shortage costs into account. While each paper considers the multiple products and multiple resources problems with demand uncertainties, their focus is limited to single-period models.

Apart from the studies which focus on flexible capacity investment, many efforts to solve generalized capacity-investment problems have also been made. Harrison and van

Mieghem (1999) develop a single-period planning model to incorporate both capacity investment and production decisions for a multiple-product manufacturing firm. Their study yields a multi-dimensional descriptive model generated from the "news-vendor model", and gives qualitative insights into real-world capacity-planning and capital-budgeting practices. Nevertheless, the decisions on optimal capacity investment are highly generalized, and the production plan decisions are not explicitly presented. van Mieghem and Rudi (2002) extend the work of Harrison and van Mieghem (1999) to include an operations environment with multiple products, production processes, storage facilities and inventory management. Moreover, they investigate how the structural properties of a single period extend to a multi-period setting. They also improve previous studies by considering some inventory-management issues.

Many studies have made extensive use of game theoretic models in the development of product pricing and competitive strategic investment models, among others. For instance, van Mieghem (1999) uses a game-theoretic approach to model the coordination process of simultaneous investment, production, and subcontracting decisions. The model's objective is to maximize the overall supply chain system profit and to analyze the size and timing of capacity investment. While capacity acquisition problems have been studied extensively, each paper mentioned above focused on single-firm operations. The competition for resources, however, is a common phenomenon in real-world operations in a multi-firm industry involving a particular product but is generally ignored in the literature because it often increases the intractability of the models, regardless of whether the model is stochastic or game theoretic.

Increasing global competition and cost pressure force businesses to discover undetected cost-saving potentials on investment in resources. Arnold et al. (2009) presents a deterministic optimal control approach optimizing the procurement and inventory policy of a company that is processing a raw material when the purchasing price, holding cost, and the demand rate fluctuate over time. However, they do not consider the effect of resource competition.

The three papers listed below address capacity decision problems emphasizing real-world capacity competition. Roller and Sickles (2000) propose a two-stage pricing and capacity-decision model considering price and capacity competition simultaneously. In the first stage, the capacity is determined and a price-setting game is performed in the second stage. Chen and Wan (2005) also study a service capacity competition problem for two make-to-order firms that are modeled as single-server queueing systems. They characterize the Nash equilibrium of the competition. The firms make their capacity choice based on the equilibria. Cheng et al. (2003) study the price and capacity competition of two application-service providers. The authors suggest that the providers with higher capacity would charge a higher price and enjoy a larger market share. Although capacity competition problems have not completely escaped notice, the aforementioned studies focus strictly on service industries modeled as queueing systems. The special operations nature of the service industry restricts the methods from being generalized to other industries, such as manufacturing or other more complicated service systems.

While great progress has been made in the development of capacity-investment models and approaches, most studies have focused on macro analysis rather than practical applications. Many complicated decision factors, such as time-varying costs and inventory management, have been left unconsidered. In this sense, lot-sizing methods can compensate perfectly for this deficiency in the game theoretic models, with the combination approach resolving the real-world capacity-investment and production problems more realistically.

Lot-sizing problems have been studied extensively for the past half century. Wagner and Whitin (1958) give a forward algorithm for a general dynamic version of the uncapacitated economic lot-sizing model. Since then, various variants, including single-item and multi-item, uncapacitated and capacitated lot-sizing problems, remain an important topic in Operations Research fields. More recent results include Federgruen and Tzur (1991), who consider a dynamic lot-sizing model with general cost structure. The authors give a simple forward algorithm which solves the general dynamic lot-size model in $O(T \log T)$ time and with $O(T)$ space requirement. This is an important improvement over the well-known shortest path algorithm solution in $O(T^2)$ space, advocated previously. Wagelmans et al. (1992) extend the range of allowable cost data to allow for coefficients that are unrestricted in sign. They developed an algorithm to solve the resulting problem in $O(T \log T)$ time.

However, the uncapacitated lot-sizing problem is an ideal case and hardly applicable to real-world operations. Capacity constraints always heavily influence production-plan decision making. Furthermore, the general capacitated lot-sizing problem is $\mathcal{NP}$-hard, see Bitran and Yanasse (1982). For the special case of a constant limit over our decision period, a number of efficient algorithms are capable of calculating an optimal production plan. For example, Florian and Klein (1971) present an algorithm with the computational complexity ($O(T^4)$) for the capacitated lot-sizing problem with constant capacity limits, exploring the important properties of an optimal production plan, the optimal plan consisting of a sequence of optimal sub-plans. Baker et al. (1978) discover some important properties of an optimal solution to the problem when the production and inventory-holding costs are constant.

Some studies have tried to relax the strict cost-structure restrictions in the algorithms reviewed above. Kirca (1990) presents a dynamic programming-based algorithm with the computational complexity of $O(T^4)$, and Shaw and Wagelmans (1998) develop a dynamic programming algorithm for the capacitated lot-size problem with general holding costs and piecewise linear production costs. The algorithm of the latter reduces the computation time to $O(T^2\overline{d})$, where $\overline{d}$ is the average demand when production cost is linear. Akbalik and Penz (2009) study a special case of the capacitated lot sizing problem (CLSP) where the production cost is assumed to be piece-wise linear with discontinuous steps. They propose an exact pseudo-polynomial dynamic programming algorithm which makes it $\mathcal{NP}$-hard in the ordinary sense.

All the studies mentioned above address capacity competition, capacity investment and lot-sizing problems individually. The implications of combining these problems are, however, rarely discussed. An exception, Atamturk and Hochbaum (2001), studies capacity acquisition, subcontracting, and lot-sizing integrally. Although their approach makes the production plan and capacity acquisition decisions simultaneously, the authors simply discuss some special cases of production and holding-cost structure. Moreover, the study still focuses on solving a series of capacitated lot-sizing problems discretely, causing the computational complexity to increase exponentially with the number of planning periods and demands. Additionally, Ahmed and Garcia (2004) study a dynamic capacity-acquisition and assignment problem in a simplified operations setting to determine the resource capacity and allocation of the resources to tasks. The study actually proposes a capacity-expansion and planning approach without considering inventory carryover and the determination of production plans.

In summary, while the progress has been made investigating the questions of capacity acquisition decisions and lot-sizing separately, few results are available that address strategies that jointly optimize capacity acquisition and lot-sizing decisions under a competition environment.

3 The Competition Model and Notation

We consider an industry with N firms, and each produces a single item. Their productions require a common resource, measured here by capacity. The capacity level purchased by a firm is assumed to be the capacity restriction in a dynamic lot-sizing setting. Examples for this include the number of trucks to lease or a scarce raw material. The firms have to purchase the capacity at the beginning of the planning horizon and can then use the capacity in each following period. The capacity must satisfy the demand constraints, and the excess capacity will be disposed of without extra disposal costs.

The production plan will be considered in a planning horizon of T periods. If the firms face a natural selling season to introduce a new model or variant, a natural choice of T arises, e.g. $T = 52$ weeks in the automobile manufacturing industry operating with a weekly production and sales schedule. Otherwise, T is chosen to be large enough to ensure that the firms' decisions pertaining to the initial periods of the planning horizon are not affected by this truncation of the planning process. We use the following indices:

i = $1, \ldots, N$, the index for each firm in the industry;

t = $1, \ldots, T$, the index for each period.

Each firm has a demand stream during the planning horizon, known only to the firm itself and following some predictable seasonality pattern (we present and discuss six common seasonality patterns in Section 6). Thus, let

d_{it} = the demand faced by firm i in period t, $i = 1, \ldots, N$, $t = 1, \ldots, T$;

β_t = the seasonality factor in period t, $t = 1, \ldots, T$;

$\overline{d}_i$ = the average demand of firm i, $i = 1, \ldots, N$;

and $d_{it} = \overline{d}_i \beta_t$.

The firms produce their goods via a process that, in principle, allows for inventory replenishment at the beginning of each period. As in standard dynamic lot-sizing problems, we assume that *fixed* as well as *variable* production costs are incurred as well as inventory carrying costs, which are proportional to each end-of-the-period inventory. We assume cost parameters may fluctuate in arbitrary ways, and they are defined as

f_{it} = the *fixed* setup cost for a production batch delivered to firm i in period t, $i = 1, \ldots, N$; $t = 1, \ldots, T$;

a_{it} = the variable production cost for a unit product in firm i in period t, $i = 1, \ldots, N$; $t = 1, \ldots, T$;

h_{it} = the inventory carrying cost for each unit of item i at the end of period t, $i = 1, \ldots, N$; $t = 1, \ldots, T$.

At the beginning of each planning horizon, each firm i selects the level of capacity to acquire, as well as a complete production schedule for the entire planning horizon to satisfy the given demand stream $\{d_{it}\}$. We denote this capacity as

C_i = the capacity acquired by firm i.

We assume that the capacity in question is traded on a spot market. The market price for a unit of capacity is relevant with the demand of the firm and its competitors for capacity. We denote the market price as p and assume it is convex in capacity. For example, a simple linear form capacity acquisition cost can be modelled as below:

$$p = \Lambda + \theta \sum_{i=1}^{N} C_i \tag{1}$$

where Λ and θ are non-negative constants known to all players.

Note that this cost of capacity p affects the profits earned by *all* firms in the industry, as the capacity acquisition is a cost factor in each firm's profit function. At the same time, the production schedule selected by firm i affect only its *own* profit measure. It is thus possible to conceptualize the competitive model as a single-stage game between N firms, in which each firm makes a single competitive choice, i.e. the capacity level to acquire in each season. The game is characterized by the cost functions below, where $-i$ refers to the competitors of the firm i in the industry.

$\pi_i(C_i|C_{-i})$ = the cost incurred by firm i under choice of capacity C_i, assuming firm i adopts an optimal dynamic lot-sizing schedule to respond to its own demand stream, and given that the competitors choose to purchase the capacities $C_{-i} = \{C_1, \ldots, C_{i-1}, C_{i+1}, \ldots, C_N\}$.

For ease of exposition, we rewrite this function as follows:

$$\begin{aligned} \pi_i(C_i|C_{-i}) &= pC_i + K_i(C_i) \\ &= C_i\left(\Lambda + \theta \sum_{i=1}^{N} C_i\right) + K_i(C_i), \quad i = 1, \ldots, N \\ &= A_i(C_i|C_{-i}) + K_i(C_i) \end{aligned} \tag{2}$$

where $A(C_i|C_{-i})$ denotes the acquisition cost of capacity, and $K_i(C_i)$, the minimum total operating costs for firm i to serve the demand under the capacity level C_i.

In the competition model, we assume that the firms know about the total number of firms N, and be able to observe the rival firms' decisions on capacity acquisition levels. In addition, the prices of the product produced by different firms are assume to be same and constant over the entire planning horizon. Therefore, the profit maximizing objective is equivalent to the cost minimizing objective. In the rest of the paper we use the cost minimizing objective function.

Knowledge of the cost function $K_i(C_i)$ is important to be able to analyze the competition model and characterize its equilibrium behavior. However, difficulty arises from the fact that the function $K_i(C_i)$ cannot be represented in a closed analytical form. We will deal with this problem in the Subsection 5.1. Under the assumption that the firm has knowledge of its function $K_i(C_i)$ and given that the firm *knows* its competitors' capacity choices C_{-i}, the best response problem (function) of the firm can then be expressed as:

$$C_{i*}(C_{-i}) = \arg\min_{C_i} \pi_i(C_i|C_{-i}) \tag{3}$$

Before providing a complete characterization of the industry's equilibrium behavior, we first analyze an individual firm's *best-response problem* in the following Section.

4 Best Response Problem

Given the capacity decisions of other firms, a firm i has to determine its own capacity acquisition level and a corresponding lot sizing plan so that the total cost is minimized. Here, the defined lot-sizing and capacity acquisition problem is the best response problem of firm i. In this section, we analyze firm i's best response problem (3), which is crucial to describe the industry equilibrium.

4.1 Formulation

In order to model the best response problem, we further define the following decision variables:

x_{it} = the production quantity of product i, $i = 1, \cdots, N$ produced in period t, $t = 1, \dots, T$;

y_{it} = $\begin{cases} 1 & x_{it} > 0 \\ 0 & \text{otherwise} \end{cases}$;

I_{it} = the inventory amount of product i, $i = 1, \cdots, N$ at the end of period t, $t = 1, \dots, T$.

This gives rise to the following formulation of the best response problem P_b of a firm:

$$\mathbf{P}_b : \pi_i(C_i|C_{-i}) = \min\left\{\sum_{t=1}^{T} (a_{it}x_{it} + h_{it}I_{it} + f_{it}y_{it}) + A_i(\mathbf{C_i}|\mathbf{C_{-i}})\right\}, \quad i = 1, \cdots, N \tag{4}$$

subject to

$$I_{it} = x_{it} - d_{it} + I_{it-1}, \quad \forall \quad t = 1, \ldots, T \tag{5a}$$

$$x_{it} \le C_i^{\max} y_{it}, \quad i = 1, \cdots, N, \quad \forall \quad t = 1, \ldots, T \tag{5b}$$

$$x_{it} \le C_i, \quad i = 1, \cdots, N, \quad \forall \quad t = 1, \ldots, T \tag{5c}$$

$$I_{i0} = I_{iT} = 0, \quad i = 1, \cdots, N \tag{5d}$$

$$x_{it} \ge 0, \quad I_{it} \ge 0, \quad y_{it} \in \{0, 1\}, \quad C_i \ge 0, \quad \forall \quad t = 1, \ldots, T, \quad i = 1, \cdots, N. \tag{5e}$$

where the objective function (4) minimizes the production and inventory-holding costs as well as the capacity acquisition costs. Constraints on the problem include: Equation (5a) ensures that inventory is balanced; Production is restricted by (5b) and (5c), where $C_i^{\max}$ is the minimum capacity which allows the optimal uncapacitated lot size plan; Constrains (5d) set initial and final inventories to zero; and the bounds of the variables are restricted by (5e). Solving the model entails *simultaneously* determining the optimal capacity, setup periods, and production amount in each order period. Capacity is assumed to be a continuous variable, meaning that capacity can be acquired at any non-negative level. If it is assumed that the firms observe their competitors capacity decisions, the best response problem can be solved according to the analysis in following subsection.

4.2 Calculation of the optimal capacity and lot sizes

The simultaneous calculation of the optimal capacity and production plan, as explained above, is a MIP model $\mathbf{P}_b$ with quadratic objective function but with all constraints as linear. This problem class is generally $\mathcal{NP}$-hard according to Garey and Johnson (1979) and Poljak and Wolkowicz (1995). While the general capacitated lot-sizing problem is $\mathcal{NP}$–hard (see Bitran and Yanasse (1982)), polynomial time algorithms are available in the special case of a *given* constant capacity. For example, Florian and Klein (1971) suggest an O(T^4) algorithm, and alternative approaches are also suggested by van Hoesel and Wagelmans (1996) and Chen et al. (1994) that run in $O(T^3)$ time. Therefore, the problem P_b can be solved by discretizing the interval of potential values for the capacities and solving for each of those values. Consequently, the best response problem is not $\mathcal{NP}$-hard in the strong sense, and in principal, it can therefore be solved in pseudo-polynomial time.

While a commercial package such as CPLEX can *in theory* handle the best response problem as described above with a quadratic objective and linear constraints, computational times on a HP 2.0 GHz with 1 GB memory were typically in excess of a few hours for our examples as described in Section 6 and the optimizer even often ran out of memory and delivered no result. Therefore, in this paper, we discretize the potential capacity space and evaluate the total cost function $\pi_i(C_i|C_{-i})$ at each point to find the optimal solution of the problem $\mathbf{P}_b$ under the assumption that the variable of capacity is continuous. For each potential capacity value, we find the best capacitated lot-sizing replenishment plan by a standard MIP solver such as CPLEX. We then pick the capacity resulting in the least sum of capacity acquisition cost and cost resulting from the capacitated lot-sizing production plan.

First, we need to determine the possible capacity range of each firm. The range is defined by an integer lower bound $C_i^{\min}$ that allows a feasible solution of the best response problem and an integer upper bound $C_i^{\max}$. They can be calculated as described below:

$$C_i^{\min} = \max_{t=1,\ldots,T} \left\{ \frac{D_i(t)}{t} \right\}, \quad i = 1, \cdot, N \tag{6}$$

where $D_i(t) = \sum_{j=1}^{t} d_{ij}$. The upper bound $C_i^{\max}$ can be determined by solving the uncapacitated lot-sizing problem, and it can be calculated by the classic Wagner-Whitin algorithm or by other algorithms proposed by Federgruen and Tzur (1991) and Wagelmans et al. (1992). $C_i^{\max}$ equals the maximum lot size over the planning horizon. If capacity increases up to $C_i > C_i^{\max}$, the lot-sizing cost is no longer decreasing, and the capacity acquisition cost is increasing. Thereby, $\pi(\mathbf{C}) > \pi(C_i^{\max})$, when $C_i > C_i^{\max}$, and the firm will not ever be better off by acquiring a capacity level $C_i > C_i^{\max}$. Therefore, it is not necessary to consider the capacity values which are greater than $C_i^{\max}$. In the remainder of the paper, we focus our analysis on the range $[C_i^{\min}, C_i^{\max}]$.

Next, given the capacity decisions of other competing firms' C_{-i}, we consider each integer capacity level $C_i = C_i^{\min}, C_i^{\min} + \triangle, C_i^{\min} + 2\triangle, \cdots, C_i^{\max}$, where $\triangle$ is assumed to be an integer such as $1, 2, \cdots$, and apply a standard solver to solve the capacitated lot-sizing problem so that the optimal capacity level and lot-sizing plan are obtained over all the capacity levels. Given the unit increment of capacity level, it is reasonable to assume the total cost of a firm is piece-wise linear function in capacity.

Upon the obtained piecewise-linear functions for each discretized capacity level, the pareto-optimal capacity is obtained when the total capacity acquisition and lot-sizing cost reaches the minimal. Furthermore, the cost function $K_i(C_i)$ has following property in Proposition 1.

Proposition 1 *Given the other competing firms' capacity decisions C_{-i}, the lot sizing and capacity acquisition cost function $K_i(C_i)$ of firm i is non-increasing and quasi-convex in its own capacity level C_i .*

Proof: With respect to the best response problem $\mathbf{P}_b$, all constraints are linear, and thus, constraint set is concave. Suppose there exist two capacity levels $C_i^1 < C_i^2$, and $C_i^1, C_i^2 \in [C_i^{\min}, C_i^{\max}]$, because $K_i(C_i)$ is non-increasing in C_i, we have

$$K_i(\alpha C_i^1 + (1-\alpha)C_i^2) \leq \max\{K_i(C_i^1), K_i(C_i^2)\} \tag{7}$$

where $\alpha \in [0,1]$, and thus, $K_i(C_i)$ is quasi-convex in C_i, since a firm's lot sizing cost will not be affected by other firms' capacity decisions, we can also see $K_i(C_i)$ is quasi-convex in its own capacity level C_i. □

We have conducted an extensive numerical study to illustrate the property. For an illustration of Proposition 1, see Figure 1 selected from a numerical example discussed in Section 6. We also present an example of total cost curve in Figure 1 over the capacity range $[C_i^{\min}, C_i^{\max}]$, which is selected from a numerical example discussed in Section 6.

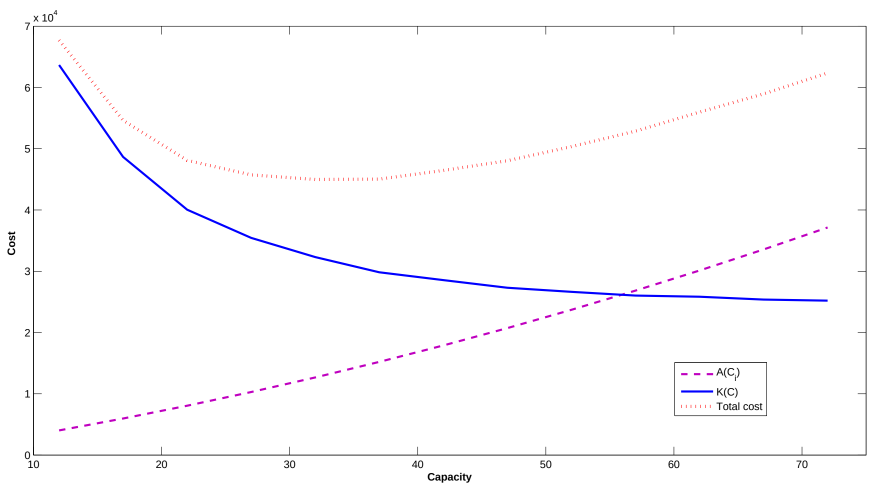

Figure 1: An illustration of convexity on cost function in capacity increase

A sufficient condition for the existence of an equilibrium is from Theorem 2.1 in Vives (1999) — originally attributed to Debreu (1952): the strategy sets are convex and compact, and the payoff to firm i is continuous in the actions of all firms and quasi-concave in its own control variable. Since we exclusively deal with cost in this paper, the profit function is just the negated cost function and hence concave in the control variable, namely the capacity choice. However, the quasi-convexity and continuity of each firm's total cost function in our capacity competition game cannot be guaranteed, and therefore, we are not able to show any explicit equilibrium results of the firms.

5 Equilibrium analysis

In order to address the competition problem, we first introduce an approximation of lot-sizing cost function $\widetilde{K}_i(C_i)$. We show that the approximation indeed results in values very close to the actual cost function and then proceed to investigate the equilibrium behavior.

5.1 Approximation of lot-sizing cost

According to Proposition 1, the lot sizing cost function $K_i(C_i)$ is non-increasing and quasi-convex, but this does not suffice to establish equilibrium behavior. Therefore, we seek to approximate the lot sizing cost function by a convex function. Ubhaya (1979) provides an algorithm to find the optimal approximation convex function of a quasi-convex function. Because the approximation process is not our focus, in analogy to Federgruen and Meissner (2009), we apply an approximation function model of the lot-sizing cost $K_i(C_i)$ as below:

$$K_i(C_i) \sim \widetilde{K}_i(C_i) = T\overline{d}_i^2\left[\eta^i + \frac{\zeta^i}{(C_i)^{\gamma^i}}\right], \quad i = 1, \ldots, N, \quad C_i \in [C^{\min}, C^{\max}]. \tag{8}$$

where $\eta^i, \zeta^i > 0$ and $\gamma^i > 0$ are appropriate constants. Since the function $K(C_i)$ has no closed form, we consider the discrete function values in the valid domain $[C_i^{min}, C_i^{max}]$, and apply the idea of least square curve fitting method to determine the parameters of the approximation function $\widetilde{K}_i(C_i)$ which minimizes the sum of squared differences between the left and the right sides of equation (8).

In order to estimate the approximation function more accurately, we calculate the constants of the approximation function based on different demand seasonality patterns and fixed setup cost levels. Assuming that firm i faces a planning horizon of T periods, and the demand behaves according to $d_{it} = \beta_t\bar{d}_i$, six seasonality patterns $\{\beta_t : t = 1, \ldots, T\}$ are typical in reality as follows. The first pattern reflects a situation where demand functions are time-invariant. The second pattern shows one with linear growth, while the third shows linear decline. The fourth and fifth patterns represent a planning horizon with a single season of peak demands either at the beginning or at the end of the planning horizon. The last pattern (VI) is cyclical with a cycle length of six periods, such that demands in the two middle periods of each cycle are 7 times their value in the first and last period, while $\beta_t = 1$ in the remaining two periods of the cycle.

(1) Time-invariant demand functions: $\beta_t = 1; \quad t = 1, \ldots, T$

(2) Linear Growth: $\beta_t = \beta_0 + (\beta_T - \beta_0)\frac{(t-1)}{T-1}; \quad t = 1, \ldots, T.$

where β_0 is the base seasonality factor. We take example of $\beta_0 = 0.25$, and then according to $\sum_{t=1}^{T} \beta_t = T$, $T = 54$, β_T can be calculated as 1.75. Therefore, we have linear growth seasonality pattern $\beta_t = 0.25 + 1.5\frac{(t-1)}{T-1}; \quad t = 1, \ldots, T$. Similarly, other parameter sets could be derived and applied, but the results of our model will not be affected.

(3) Linear Decline: $\beta_t = \beta_T - (\beta_T - \beta_0)\frac{(t-1)}{T-1}; \quad t = 1, \ldots, T.$

Analogous with seasonality pattern (II), let $\beta_0 = 0.25$, and we obtain $\beta_t = 1.75 - 1.5\frac{(t-1)}{T-1}; \quad t = 1, \ldots, T$.

(4) Holiday Season at the Beginning of the Planning Horizon:

$$\beta_t = \begin{cases} \beta_0 + \frac{\mathcal{P}\beta_0}{\mathcal{L}/2-1w}(t-1) & , t = 1, \ldots, \mathcal{L}/2 \\ (1+\mathcal{P})\beta_0 - \frac{\mathcal{P}\beta_0}{(\mathcal{L}/2-1)}(t-1-\mathcal{L}/2) & , t = \mathcal{L}/2+1, \ldots, 12 \\ \beta_0 & , t = \mathcal{L}+1, \ldots, T \end{cases} \tag{9}$$

where $\mathcal{L}$ represents the length of the peak season, and $\mathcal{P}$ describes the degree of peak seasonality factor over base, and for example, if $\mathcal{P} = 10$, the highest demand is 10 times of the base demand. Based on the condition $\sum_{t=1}^{T} \beta_T = T$, we have $\beta_0 = \frac{T}{T+\mathcal{L}\frac{\mathcal{P}}{2}}$. Let $\mathcal{L} = 12$, $\mathcal{P} = 10$; we obtain the exact holiday seasonality pattern formula as below.

$$\beta_t = \begin{cases} \frac{54}{114} + \frac{540}{570}(t-1) & , t = 1, \ldots, 6 \\ \frac{594}{114} - \frac{540}{570}(t-7) & , t = 7, \ldots, 12 \\ \frac{54}{114} & , t = 13, \ldots, T \end{cases} \tag{10}$$

(5) Holiday Season at the End of the Planning Horizon:

$$\beta_t = \begin{cases} \beta_0 & , t = 1, \ldots, T-\mathcal{L} \\ \beta_0 + \frac{\mathcal{P}\beta_0}{(\mathcal{L}/2-1)}(t-T+\mathcal{L}-1) & , t = T-\mathcal{L}+1, \ldots, T-\mathcal{L}/2 \\ (1+\mathcal{P})\beta_0 - \frac{\mathcal{P}\beta_0}{(\mathcal{L}/2-1)}(t-T+\mathcal{L}/2-1) & , t = T-\mathcal{L}/2+1, \ldots, T \end{cases} \tag{11}$$

Similarly with seasonality pattern (IV), we apply the seasonality pattern (V) as follows:

$$\beta_t = \begin{cases} \frac{54}{114} & , t = 1, \ldots, 42 \\ \frac{54}{114} + \frac{540}{570}(t-43) & , t = 43, \ldots, 48 \\ \frac{594}{114} - \frac{540}{570}(t-49) & , t = 49, \ldots, T \end{cases} \tag{12}$$

(6) Cyclical Pattern:

$$\beta_t = \begin{cases} \beta_0 + \frac{(\mathcal{P}-1)\beta_0}{\mathcal{L}/2-1}(t-1) & , t = 1, \ldots, \mathcal{L}/2 \\ \mathcal{P}\beta_0 - \frac{(\mathcal{P}-1)\beta_0}{\mathcal{L}/2-1}(t-\mathcal{L}/2-1) & , t = \mathcal{L}/2+1, \ldots, \mathcal{L} \\ \beta_{t\mathrm{mod}6} & , t = \mathcal{L}+1, \ldots, T \end{cases} \tag{13}$$

where $t$mod6 denotes t modulo 6. In this case, $\mathcal{L}$ represents the cycle length, and $\mathcal{P}$ denotes the multiplier of peak demand over base demand. Let $\mathcal{L} = 6$, $\mathcal{P} = 7$ in this paper, and then it is calculated $\beta_0 = 0.25$. we obtain

$$\beta_t = \begin{cases} 0.25 + 0.75(t-1) & , t = 1, \ldots, 3 \\ 1.75 - 0.75(t-4) & , t = 4, \ldots, 6 \\ \beta_{t\mathrm{mod}6} & , t = 7, \ldots, T \end{cases} \tag{14}$$

Assuming that the setup, production and inventory costs, and capacity acquisition costs are identical, the values of $K_i(C_i)$ are displayed as a function of feasible capacity levels for the six demand patterns (DP) in Figure 2a and 2b.

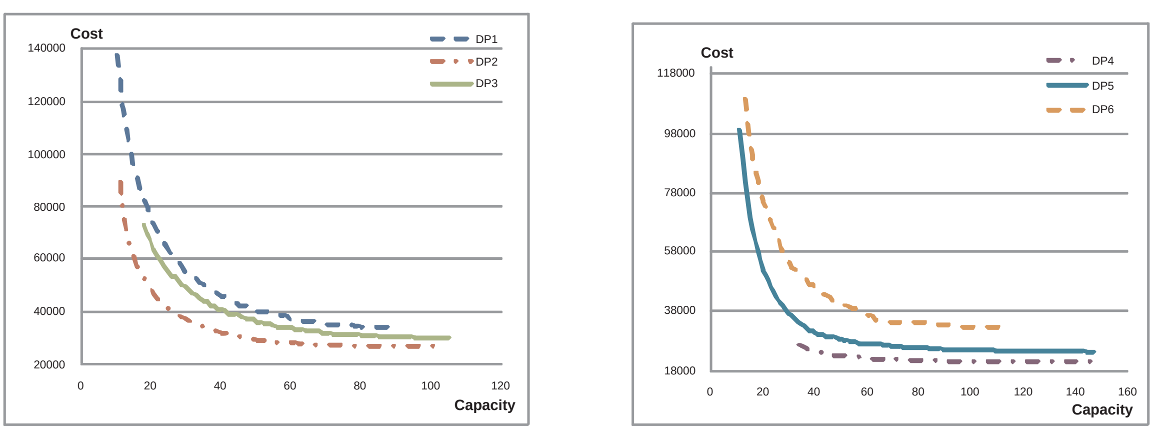

Figure 2: $K_i(C_i)$ as function of capacity in the six demand patterns

Not surprisingly, the total cost curves differ in demand seasonality pattern. Therefore, it is necessary to estimate the constants of the approximation function with respect to each seasonality pattern. Furthermore, we distinguish three levels of the fixed setup cost (expressed by Time Between Orders (TBO), see further definition in Section 6) and investigate the final form of an approximation function.

We first pick production cost $a_{it} = 15$; inventory holding cost $h_{it} = 5$; and TBO=[High, Medium, Low] for three firms $i = 1, 2, 3$, $t = 1, \cdots, T$; and average demand $d_i = [12, 10, 8]$ for three firms $i = 1, 2, 3$. Given the identical cost parameters and seasonality patterns, the firms have the same lot-sizing costs $K_i(C_i)$. Similarly, for the time-varying production and inventory holding cost cases, the approximation curve can also be estimated. Table 1 below exhibits the parameters generating the best possible fit:

TBO	Demand Pattern	$\bar{d}_i = 12$				$\bar{d}_i = 10$				$\bar{d}_i = 8$			
		γ_i	η_i	ζ_i	Gap	γ_i	η_i	ζ_i	Gap	γ_i	η_i	ζ_i	Gap
Low	1	0.13	0.00	2.44	0.91%	0.13	0.00	3.46	1.04%	0.14	0.00	4.94	1.18%
	2	5.43	1.56	1872426.80	2.02%	6.47	2.24	13947232.29	1.15%	5.40	3.19	431745.19	2.15%
	3	0.98	1.51	2.11	0.04%	0.89	2.14	2.10	0.03%	3.73	3.05	108125.05	0.24%
	4	4.53	1.51	3817067.85	0.20%	4.64	2.14	3817067.77	0.20%	2.05	2.76	698.58	0.61%
	5	2.07	1.38	751.10	0.55%	2.06	1.93	754.01	0.55%	2.05	2.76	698.58	0.61%
	6	5.45	1.55	633533.15	0.81%	5.37	2.27	336052.83	0.72%	5.45	3.18	150305.24	0.86%
Average					0.76%				0.62%				0.94%
Medium	1	1.34	2.19	90.60	1.03%	1.35	3.16	106.11	1.08%	1.33	4.59	113.53	1.15%
	2	2.54	2.55	2671.83	1.38%	2.43	3.68	1937.12	1.21%	2.55	5.43	2221.59	1.47%
	3	1.91	2.39	460.90	0.51%	1.97	3.48	573.76	0.51%	1.70	4.89	277.53	0.74%
	4	2.76	2.44	13975.71	0.21%	2.78	3.53	13547.23	0.23%	2.76	5.17	10457.31	0.23%
	5	1.94	2.33	891.18	1.16%	1.94	3.37	954.06	1.21%	1.94	4.93	928.38	1.20%
	6	1.50	2.21	143.62	0.98%	1.52	3.28	184.11	0.88%	1.50	4.65	178.60	1.07%
					0.88%				0.85%				0.98%
High	1	1.24	2.77	201.22	1.27%	1.25	4.06	244.10	1.24%	1.24	5.94	273.13	1.30%
	2	1.61	3.26	578.43	1.02%	1.58	4.73	601.15	1.05%	1.63	7.06	700.54	1.03%
	3	1.53	3.08	473.31	0.99%	1.57	4.53	604.37	0.98%	1.57	6.73	637.26	1.22%
	4	1.95	3.25	2365.37	0.53%	1.94	4.75	2447.68	0.54%	1.95	7.00	2439.81	0.55%
	5	1.64	3.20	782.33	1.27%	1.64	4.67	875.05	1.30%	1.64	6.89	926.30	1.27%
	6	1.41	2.97	331.14	2.04%	1.40	4.40	384.66	1.98%	1.42	6.40	426.27	2.08%
					1.19%				1.18%				1.24%

Table 1: Approximating curves for demand patterns (I) to (VI)

The columns of “Gap” in Table 1 display the average relative difference between the exact and the approximate curve. The narrow gaps indicate that approximations of the type (8) are very close for *any* combination of fixed setup cost levels and seasonality patterns. In addition, the performance of the approximation function is relevant with the capacity incremental value Δ. The smaller of Δ, the closer the approximation function is to the actual lot sizing and capacity acquisition cost curve.

5.2 Existence of Equilibrium, Uniqueness and Convergence

Treating the capacity C_i as continuous variable, and substituting the cost function $K_i(C_i)$ by the close approximation function (8), the total lot-sizing and capacity acquisition cost function $\pi_i(C_i|C_{-i})$ can be expressed as

$$\begin{aligned}\pi_i(C_i|C_{-i}) \sim \tilde{\pi}_i(C_i|C_{-i}) &= \tilde{K}_i(C_i) + A_i(C_i|C_{-i}) \\ &= T\overline{d}_i^2\left[\eta^i + \frac{\zeta^i}{(C_i)^{\gamma^i}}\right] + (\Lambda + \theta\sum_{i=1}^{n} C_i)C_i, \quad i = 1, \cdots, N.\end{aligned} \tag{15}$$

According to the function (15), we show that

(1) Nash equilibrium exists for the competition game;

(2) The Nash equilibrium is unique, and,

(3) The equilibrium converges by an iterative Tatônnement scheme and can be computed efficiently.

Theorem 1 *A Nash equilibrium exists for the competition game.*

Proof: Since

$$\frac{\partial^2 \tilde{\pi}_i(C_i|C_{-i})}{\partial C_i^2} = T\overline{d}_i^2 \frac{\gamma^i(\gamma^i+1)\zeta^i}{(C_i)^{\gamma^i+2}} + 2\theta \geq 0, \quad i = 1, \cdots, N. \tag{16}$$

Therefore, the approximation payoff function is continuous in the actions of all firms and convex in its own control variable. In addition, the strategy sets are convex and compact, so the conclusion holds. □

Theorem 2 *The (approximate) cost functions $\tilde{\pi}_i(C_i|C_{-i})$ satisfies the dominance condition*

$$\frac{\partial^2 \tilde{\pi}_i(C_i|C_{-i})}{\partial C_i^2} \geq \frac{\partial^2 \tilde{\pi}_i(C_i|C_{-i})}{\partial C_i \partial C_j}, \quad i \neq j, \quad i, j = 1, \cdots, n \tag{17}$$

Therefore, the Nash equilibrium is unique.

Proof:

$$\frac{\partial \tilde{\pi}_i(C_i|C_{-i})}{\partial C_i} = \frac{\partial \widetilde{K}_i(C_i)}{\partial C_i} + \Lambda + \theta \sum_{i=1}^{N} C_i + \theta C_i, \quad i = 1, \cdots, N \tag{18}$$

Since $\frac{\partial^2 \widetilde{K}_i(C_i)}{\partial C_i^2} = T\overline{d}^2 \frac{\gamma^i(\gamma^i+1)\zeta^i}{(C_i)^{\gamma^i+2}}$, and $\frac{\partial^2 \widetilde{K}_i(C_i)}{\partial C_i \partial C_j} = 0$, we have

$$\begin{aligned} \frac{\partial^2 \tilde{\pi}_i(C_i|C_{-i})}{\partial C_i^2} &= T\overline{d}^2 \frac{\gamma^i(\gamma^i+1)\zeta^i}{(C_i)^{\gamma^i+2}} + 2\theta \geq 0 \\ \frac{\partial^2 \tilde{\pi}_i(C_i|C_{-i})}{\partial C_i \partial C_j} &= \theta \geq 0, \quad i \neq j, \quad i, j = 1, \cdots, N \end{aligned} \tag{19}$$

Thus, $\frac{\partial^2 \tilde{\pi}_i(C_i|C_{-i})}{\partial C_i^2} \geq \frac{\partial^2 \tilde{\pi}_i(C_i|C_{-i})}{\partial C_i \partial C_j}$, and a unique equilibrium is guaranteed. □

Furthermore, the unique equilibrium can be efficiently computed as the limit point of the following simple Tatônnement scheme:

Tatônnement scheme: Starting with an arbitrary capacity vector $\mathbf{C}^{(0)}$, in the k^{th} iteration of the scheme, each firm determines it capacity $C_i^{(k)}$ and lot sizes which solves the best response problem (4). Assume all competing firms' capacities are set according to their value in the capacity vector $\mathbf{C}^{(k-1)}$.

Convergency of this Tatônnement scheme is guaranteed when the game is supermodular. Theorem 2 shows that the supermodularity property holds because

$$\frac{\partial^2 \tilde{\pi}_i(C_i|C_{-i})}{\partial C_i \partial C_j} = \theta \geq 0, \quad \forall i = 1, \cdots, N. \tag{20}$$

6 Numerical example

A complete numerical experiment investigating our approach to make capacity acquisition and production decisions with multi-firm capacity competition is performed in this section. Using different combinations of demand pattern and TBO, we generate a number (18) of hypothetical test problems. The algorithm on the competition game is also coded using MatLab and calls for the approximation function of the best response problem of each firm. The problem instances are solved on a Pentium 4 PC with 1G RAM.

Three firm games are set based on the following data: demand $\overline{d}_i = (8, 10, 12)$, production cost $a_i = (17, 15, 13)$ and inventory holding cost $h_i = (6, 5, 4)$. For the sake of simplicity and replicability, we use the constant production and inventory holding costs in the test problems, but it will not be much more difficult to solve the problems with time varying costs.

Given that the firms produce an identical product, the demand patterns in each game instance are identical. The hypothetical test problems vary with the combination of the different demand patterns and fixed setup cost levels. We determine the fixed setup cost *indirectly* by first choosing the EOQ-cycle time "Time-between-Orders (TBO)" $= \sqrt{\frac{2f}{h\bar{d}}}$ and determine the fixed setup cost f_i from this identity. Let TBO value be 2 for *low* level fixed setup cost, 5 for *medium* level fixed setup cost and 8 for *high* level fixed setup cost. Finally, let the fixed capacity acquisition cost $\Lambda = 250$ and variable capacity acquisition cost $\theta = 3$.

Based on the approximation results, we calculate each firm's best response capacity, cost and lot-sizing plan iteratively according to the Tatônnement scheme, and reach the equilibrium until not a single firm will deviate further from its decision. The computational results are presented in Table 2.

Test Problem	Demand Pattern	TBO Level	Capacity Equilibrium	Cost Equilibrium ($x10^4$)	Setups
(1)	(2)	(3)	(4)	(5)	(6)
1	DP1		[17;21;25]	[1.61;1.79;1.85]	[27;27;27]
2	DP2		[29;36;43]	[1.93;2.15;2.33]	[25;24;24]
3	DP3	[L, L, L]	[29;36;43]	[2.64;2.18;2.32]	[24;24;24]
4	DP4		[43;53;64]	[3.08;3.61;3.97]	[23;23;23]
5	DP5		[43;53;64]	[2.78;3.14;3.35]	[24;24;24]
6	DP6		[27;35;40]	[1.77;2.04;2.11]	[19;19;19]
7	DP1		[41;51;61]	[3.58;3.94;4.12]	[11;11;11]
8	DP2		[56;70;83]	[3.73;4.16;4.31]	[11;11;11]
9	DP3	[M, M, M]	[56;70;83]	[3.76;4.18;4.36]	[10;10;10]
10	DP4		[119;148;178]	[5.73;6.54;6.99]	[10;9;10]
11	DP5		[77;96;115]	[4.55;5.05;5.26]	[10;10;10]
12	DP6		[51;73;76]	[3.76;4.20;4.31]	[9;9;9]
13	DP1		[65;81;97]	[5.94;6.53;6.79]	[7;7;7]
14	DP2		[82;102;122]	[6.18;6.88;7.09]	[7;7;7]
15	DP3	[H, H,H]	[95;106;127]	[6.27;6.99;7.25]	[6;6;6]
16	DP4		[120;150;179]	[7.34;8.22;8.63]	[6;6;6]
17	DP5		[180;224;269]	[8.31;9.10;9.39]	[6;6;6]

Continued on next page–

Table 2 – continued from previous page

Test Problem	Demand Pattern	TBO Level	Capacity Equilibrium	Cost Equilibrium ($x10^4$)	Setups
(1)	(2)	(3)	(4)	(5)	(6)
18	DP6		[89;112;133]	[6.35;7.13;7.27]	[6;8;7]
19	DP1		[17;51;97]	[1.85;4.01;6.01]	[27;11;7]
20	DP2		[29;70;122]	[2.36;4.23;6.28]	[25;11;7]
21	DP3	[L,M,H]	[29;70;127]	[3.38;4.28;6.27]	[24;10;6]
22	DP4		[43;148;179]	[4.78;5.77;7.68]	[23;9;6]
23	DP5		[43;96;269]	[3.98;5.85;6.98]	[24;10;6]
24	DP6		[27;73;133]	[2.18;4.40;6.31]	[19;9;7]
25	DP1		[17;81;61]	[1.84;5.78;4.16]	[27;7;11]
26	DP2		[29;102;83]	[2.33;6.09;4.34]	[25;7;11]
27	DP3	[L,H,M]	[29;106;83]	[3.33;6.04;4.42]	[24;6;10]
28	DP4		[43;150;178]	[4.79;7.43;6.10]	[23;6;10]
29	DP5		[43;224;115]	[3.86;6.75;5.96]	[24;6;10]
30	DP6		[27;112;76]	[2.13;6.09;4.41]	[19;8;9]
31	DP1		[41;21;97]	[3.61;2.08;5.95]	[11;27;7]
32	DP2		[56;36;122]	[3.75;2.64;6.21]	[11;24;7]
33	DP3	[M,L,H]	[56;36;127]	[3.81;2.78;6.20]	[10;24;6]
34	DP4		[119;53;179]	[4.96;5.55;7.45]	[10;23;6]
35	DP5		[77;53;269]	[5.17;4.51;6.88]	[10;24;6]
36	DP6		[51;35;133]	[3.85;2.51;6.17]	[9;19;7]
37	DP1		[41;81;25]	[3.54;5.66;2.16]	[11;7;27]
38	DP2		[56;102;43]	[3.69;5.97;2.83]	[11;7;24]
39	DP3	[M,H,L]	[56;106;43]	[3.74;5.92;2.94]	[10;6;24]
40	DP4		[119;150;64]	[4.82;7.04;6.05]	[10;6;23]
41	DP5		[77;224;64]	[4.98;6.59;4.68]	[10;6;24]
42	DP6		[51;112;40]	[3.77;5.98;2.57]	[9;8;19]
43	DP1		[65;21;61]	[5.19;2.04;4.08]	[7;27;11]
44	DP2		[82;36;83]	[5.42;2.58;4.25]	[7;24;11]
45	DP3	[H,L,M]	[95;36;83]	[5.43;2.75;4.39]	[6;24;10]
46	DP4		[120;53;178]	[6.55;5.55;5.86]	[6;23;10]
47	DP5		[180;53;115]	[6.07;4.24;5.72]	[6;24;10]
48	DP6		[89;35;76]	[5.37;2.43;4.31]	[6;19;9]
49	DP1		[65;51;25]	[5.14;3.87;2.13]	[7;11;27]
50	DP2		[82;70;43]	[5.37;4.07;2.80]	[7;11;24]
51	DP3	[H,M,L]	[95;70;43]	[5.38;4.17;2.95]	[6;10;24]
52	DP4		[120;148;64]	[6.39;5.38;6.03]	[6;9;23]
53	DP5		[180;96;64]	[6.01;5.41;4.53]	[6;10;24]

Continued on next page–

Table 2 – continued from previous page

Test Problem	Demand Pattern	TBO Level	Capacity Equilibrium	Cost Equilibrium ($x10^4$)	Setups
(1)	(2)	(3)	(4)	(5)	(6)
54	DP6		[89;73;40]	[5.39;4.21;2.57]	[6;9;19]

Table 2: The equilibrium solution of the competition games

In Table 2, Column (3) shows the possible fixed setup cost levels for the three firms. We consider the instances when the firms have identical TBO levels and different TBO levels respectively. The results show that the firms choose similar production strategy when the fixed setup costs are at the same level. However, when fixed setup costs differ from each other, the firms choose rather different setup policy. In addition, Column 4 represents the firms' decisions on capacity, and column 5 shows the total costs of the firms. Column 6 shows the setup numbers of the firms. In general, all instances converge within a small number of iterations.

7 Conclusion

This paper considers a multiple firm lot-sizing problem with resource competition. We model and solve the competition game and discuss the equilibrium behaviors of the firms. As a best response problem of a firm, a typical capacity acquisition and lot-sizing problem is solved by line search. The algorithm solves the capacity acquisition, production, and inventory decisions simultaneously for multiple firms iteratively. In order to tackle the complexity of dynamic lot sizing problem and potential discontinuity of its cost function, a close approximation is applied to substitute the dynamic lot sizing cost. Under the mild conditions, we show the existence and uniqueness of equilibrium, and furthermore, the equilibrium converges within finite iterations of computation. In addition, the extension of multiple products share a common resource can be easily adapted into our method by solving the approximation problem of multiple product lot sizing problem.

In the present study, we only consider a rather simple structure of the resource competition and dynamic lot sizing problem. First, the analysis is limited to the deterministic supply and demand. This leaves the future research opportunities on the capacity acquisition and competition problem under random supply and demand uncertainty. In addition, we only consider a constant capacity setting over the planning horizon. It would also be interesting to analyze the time varying capacity situation. If the capacity can be purchased or disposed of in each period, it could lead to a solution for a dynamic competition game and the problem would be much more complicated.

References

Ahmed S., Garcia R., 2004. Dynamic capacity acquisition and assignment under uncertainty. *Annals of Operations Research*, 124(1-4), 267–283.

Akbalik A., Penz B., 2009. Exact methods for single-item capacitated lot sizing problem with alternative machines and piece-wise linear production costs. *International Journal of Production Economics*, 119(2), 367–379.

Arnold J., Minner S., Eidam B., 2009. Raw material procurement with fluctuating prices. *International Journal of Production Economics*, 121(2), 353–364.

Atamturk A., Hochbaum D.S., 2001. Capacity acquisition, subcontracting, and lot sizing. *Management Science*, 47(8), 1081–1100.

Baker K.R., Dixon P., Magazine M.J., Silver E.A., 1978. An algorithm for the dynamic lot-size problem with time-varying production capacity constraints. *Management Science*, 24(16), 1710–1720.

Bitran G.R., Yanasse H.H., 1982. Computational complexity of the capacitated lot size problem. *Management Science*, 28(10), 1174–1186.

Chen H., Wan Y.W., 2005. Capacity competition of make-to-order firms. *Operations Research Letters*, 33(2), 187–194.

Chen H.D., Hearn D.W., Lee C.Y., 1994. A new dynamic programming algorithm for the single item capacitated dynamic lot size model. *Journal of Global Optimization*, 4, 285–300.

Cheng H.K., Demirkan H., Koehler F.J., 2003. Price and capacity competition of application services duopoly. *Information Systems and e-Business Management*, 1(3), 305–329.

Debreu G., 1952. A social equilibrium existence theorem. *in Proceedings of the National Academy of Sciences*, 38, 886–893.

Federgruen A., Meissner J., 2009. Competition under time-varying demands and dynamic lot sizing costs. *Naval Research Logistics*, 67(1), 57–73.

Federgruen A., Tzur M., 1991. A simple forward algorithm to solve general dynamic lot sizing models with n periods in $o(n \log n)$ or $o(n)$ time. *Management Science*, 37(8), 909–925.

Fine C.H., Freund R.M., 1990. Optimal investment in production-flexible manufacturing capacity. *Management Science*, 36(4), 449–446.

Florian M., Klein M., 1971. Deterministic production planning with concave costs and capacity constraints. *Management Science*, 18(1), 12–20.

Garey M.R., Johnson D.S., 1979. *Computers and intractability—A guide to the theory of NP-Completeness.* W H Freeman and Company.

Harrison J.M., van Mieghem J.A., 1999. Multi-resource investment strategies: operational hedging under demand uncertainty. *European Journal of Operational Research*, 113(1), 17–29.

Kirca O., 1990. An efficient algorithm for the capacitated single item dynamic lot size problem. *European Journal of Operational Research*, 45(1), 15–24.

Netessine S., Dobson G., Shumsky R.A., 2002. Flexible service capacity: optimal investment and the impact of demand correlation. *Operations Research*, 50(2), 375–388.

Poljak S., Wolkowicz H., 1995. Convex Relaxations of (0, 1)-Quadratic Programming. *Mathematics of Operations Research*, 20(3), 550–561.

Roller L.H., Sickles R., 2000. Capacity and product market competition: measuring market power in a ‘puppy-dog’ industry. *International Journal of Industrial Organization*, 18(6), 845–865.

Shaw D.X., Wagelmans A.P.M., 1998. An algorithm for single-item capacitated economic lot sizing with piecewise linear production costs and general holding costs. *Management Science*, 44(6), 831–838.

Ubhaya V.A., 1979. An $O(n)$ algorithm for discrete n-point convex approximation with applications to continuous case. *Journal of Mathematical Analysis and Application*, 72, 338–354.

van Hoesel C.P.M., Wagelmans A.P.M., 1996. An $\mathrm{O}(\mathrm{T}^3)$ algorithm for the economic lot-sizing problem with constant capacities. *Management Science*, 42(1), 142–150.

van Mieghem J.A., 1998. Investment strategies for flexible resources. *Management Science*, 44(7), 1071–1078.

——, 1999. Coordinating investment, production and subcontracting. *Management Science*, 45(7), 954–971.

van Mieghem J.A., Rudi N., 2002. Newsvendor networks: inventory management and capacity investment with discretionary activities. *Manufacturing & Service Operations Management*, 4(4), 313–335.

Vives X., 1999. *Oligopoly pricing: Old ideas and new tools*. Cambridge, MA: The MIT Press.

Wagelmans A., van Hoesel S., Kolen A., 1992. Economic lot sizing: an $O(n \log n)$algorithm that runs in linear time in the Wagner-Whitin case. *Operations Research*, 40(S1), S145–S156.

Wagner H.M., Whitin T.M., 1958. Dynamic version of the economic lot size model. *Management Science*, 5(1), 89–96.