Lancaster University
MANAGEMENT SCHOOL

Dr Joern Meissner
Department of Management Science

PhD Course – Foundations of Optimization

Module Leader and Lecturer

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Time and Place

January 19th, 2009, Monday, 04–06pm, Meeting Room3
January 21st, 2009, Wednesday, 04–06pm, Meeting Room3
January 23rd, 2009, Friday, 04–06pm, Meeting Room3
January 24th, 2009, Saturday, 04–06pm, Meeting Room3
January 26th, 2009, Monday, 04–06pm, Meeting Room3
January 28th, 2009, Wednesday, 04–06pm, Meeting Room3
January 30th, 2009, Friday, 04–06pm, Meeting Room3
January 31st, 2009, Saturday, 04–06pm, Meeting Room3

Course Website

The web page for the course can be found on the Lancaster bulletin board.

Course Description

Mathematical optimization provides a unifying framework for studying issues of rational decision-making, optimal design, effective resource allocation and economic efficiency. It is therefore a central methodology of many business-related disciplines, including operations research, marketing, accounting, economics, game theory and finance. In many of these disciplines, a solid background in optimization theory is essential for doing research.

This course provides a rigorous introduction to the fundamental theory of optimization. It examines optimization theory in two primary settings: optimization in R^n and optimization over time (optimal control and dynamic programming). The course emphasizes the unifying themes (convexity, duality, Lagrangian multipliers) that are common to all these areas of mathematical optimization. Applications from problem areas in which optimization plays a key role is also introduced. The goal of the course is to provide students with a foundation sufficient to use basic optimization in their own research work and/or to pursue more specialized studies involving optimization theory.

The course is open to all students, but it is designed for entering doctoral students. The prerequisites are calculus, linear algebra and some familiarity with real analysis. Other concepts are developed as needed throughout the course.

Course Assessment

The assessment for this course consists of homework and a final exam:

- Homework is collected regularly during the course.
- A problem set at the end of the course (individual take-home exam).

Reading and Lecture Notes

I will mainly follow these texts:

- Sundaram, R.K. *A First Course in Optimization Theory*. Cambridge University Press 1996. While the library has some copies on reserve, I would recommend that you purchase this useful and moderately priced book.
- Sethi, Suresh P., and G. Thompson. *Optimal Control Theory: Applications to Management Science and Economics*. Kluwer Academic Publishers, 2000.

The remainder of the course is self-contained and I will distribute a lecture pack.

Additional optional reading

In addition to the book above, you may consult the following texts depending on your interests:

- Bertsekas, D. *Nonlinear Programming*, Second Edition, Athena Scientific Publishing, 1999.
- Bertsekas, D. *Dynamic Programming and Optimal Control*, Second Edition, Athena Scientific Publishing, 2001.
- Bazaraa, M., Sherali, H., and Shetty, C. *Nonlinear Programming: Theory and Algorithms*, Second Edition, John Wiley & Sons, 1993.

Acknowledgements

This course is based on the course I took myself as a doctoral candidate with Garrett van Ryzin at Columbia Business School. Thanks to Sergei Savin for sharing the latest incarnation of this course with me.

Detailed course outline

1. Classical optimization
    a. Unconstrained optimization: Weierstrass Theorem, necessary and sufficient conditions for the unconstrained local optima
    b. Constrained optimization: Theorem of Lagrange, KKT and Fritz-John conditions
    c. Convex sets and convex functions, implications of convexity for optimization
    d. Generalizations of convexity
    e. Duality in optimization problems: Duality Theorem, LP and QP duality, properties of dual problems, duality for convex problems
    f. Parametric continuity and monotonicity
2. Dynamic Programming
    a. Finite-Horizon Dynamic Programming: Markovian Strategies, existence of optimal strategy
    b. Stationary Discounted Dynamic Programming: the Bellman equation, existence of optimal strategy
3. Dynamic Optimization
    a. Calculus of variations
    b. Optimal control: general formulation, first and second-order conditions, bang-bang control, Pontryagin's Maximum Principle