



## Lecture 3

- Understanding the optimizer sensitivity report
  - ▶ Shadow (or dual) prices
  - ▶ Right hand side ranges
  - ▶ Objective coefficient ranges
- Bidding Problems
- Summary and Preparation for next class

## Sensitivity Analysis: Shadow (or Dual) Prices

- Because data are usually never known precisely, we often would like to know: How does the optimal solution change when the LP data changes, i.e., how *sensitive* is the optimal solution to the data?
- Or phrased another way, how much would the management of Shelby be willing to pay to increase the capacity of the Model *S* assembly department by 1 unit, i.e., from 1900 to 1901?

- Shelby Shelving Linear Program

$$\max 260 S + 245 LX - 385,000 \quad (\text{Net Profit})$$

subject to:

$$\text{(S assembly)} \quad S \leq 1900$$

$$\text{(LX assembly)} \quad LX \leq 1400$$

$$\text{(Stamping)} \quad 0.3 S + 0.3 LX \leq 800$$

$$\text{(Forming)} \quad 0.25 S + 0.5 LX \leq 800$$

$$\text{(Non-negativity)} \quad S, LX \geq 0$$

- Optimal solution:  $S = 1900$ ,  $LX = 650$ , Net Profit = \$268,250.



## Shadow Price

○ Answer: **NO**

- ▶ Because producing 1 more Model *S* would require an additional 0.25 hours in the forming department (which is currently used at full capacity). Hence, producing 1 more Model *S* would require a cut in Model *LX* production. To offset the extra 0.25 hours on the forming machine, Model *LX* production must be cut by 0.5 units.

- ▶ Recall: Shelby Shelving Linear Program

$$\max 260 S + 245 LX - 385,000 \quad (\text{Net Profit})$$

subject to:

$$\text{(S assembly)} \quad S \leq 1900$$

$$\text{(LX assembly)} \quad LX \leq 1400$$

$$\text{(Stamping)} \quad 0.3 S + 0.3 LX \leq 800$$

$$\text{(Forming)} \quad 0.25 S + 0.5 LX \leq 800$$

$$\text{(Non-negativity)} \quad S, LX \geq 0$$

- ▶ Optimal solution:  $S = 1900$ ,  $LX = 650$ ,  $\text{Net Profit} = \$268,250$ . Stamping hours used: 765. Forming hours used: 800.

## Shadow Price (continued)

Analysis of the change in profit:

Increase in S by 1 unit:	+\$260
Decrease in LX by 1/2 unit:	-\$245(0.5)= -\$122.5
Change in net profit:	+\$260 -\$122.5= +\$137.5

- Shadow Price for Model S assembly constraint:

$$\text{Shadow Price} = \frac{\text{Change in optimal net profit}}{\text{Change in RHS}} = 137.5$$

(RHS is short for right hand side).

- Equivalently, we can write

$$\text{Change in profit} = \text{Shadow Price} \times \text{Change in RHS}$$

- For example, an increase in Model S assembly capacity from 1900 to 1902 would be worth

$$275 = 137.5 \times 2.$$

Alternatively, a decrease in Model S assembly capacity from 1900 to 1897 would be worth

$$- 412.5 = 137.5 \times (- 3),$$

i.e., would *reduce* profit by 412.5.

# Spreadsheet Sensitivity Report

Microsoft Excel 8.0 Sensitivity Report  
Worksheet: [shelby.xls]Model  
Report Created: 8/17/98 8:17:10 PM

## Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$4	Production per month Model S	1900	0	260	1E+30	137.5
\$D\$4	Production per month Model LX	650	0	245	275	245

## Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$15	Model S assembly Used	1900	137.5	1900	233.33333	1500
\$E\$16	Model LX assembly Used	650	0	1400	1E+30	750
\$E\$17	Stamping (hours) Used	765	0	800	1E+30	35
\$E\$18	Forming (hours) Used	800	490	800	58.333333	325

- The spreadsheet optimizer's sensitivity report gives shadow-price information. Shadow prices of non-negativity constraints are often called *reduced costs*. This information is created automatically (i.e., without extra computational effort) when the LP is solved *as long as* "Assume Linear Model" is checked in the Solver Options dialog box.
- See the section "Report files and dual prices" in the reading *An Introduction to Spreadsheet Optimization Using Excel* for more information about creating reports using the Excel optimizer.

## Right-hand-Side Ranges

- The sensitivity report also gives right-hand-side ranges specified as allowable increase and allowable decrease:

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$4	Production per month Model S	1900	0	260	1E+30	137.5
\$D\$4	Production per month Model LX	650	0	245	275	245

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$15	Model S assembly Used	1900	137.5	1900	233.33333	1500
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\$E\$17	Stamping (hours) Used	765	0	800	1E+30	35
\$E\$18	Forming (hours) Used	800	490	800	58.333333	325

- The sensitivity report indicates that the shadow price for Model S assembly, 137.5, is valid for RHS ranging from  
 $1900 - 1500$  to  $1900 + 233.33$  .  
 i.e., for Model S assembly capacity from  
 $400$  to  $2133.33$  .
- In other words, the equation  
 $\text{Change in profit} = \text{Shadow Price} \times \text{Change in RHS}$ .  
 is only valid for Changes in RHS from  $-1500$  to  $+233.33$ .





## Shadow Price (continued)

- Answer: **Nothing**

- ▶ They would not be willing to pay *anything*. Why? The capacity is 1400, but they are only producing 650 Model *LX* shelves. There are already 750 units of unused capacity (i.e., *slack*), so an additional unit of capacity is worth 0. So the shadow price of the Model *LX* assembly constraint is 0.

- ▶ Recall: Shelby Shelving Linear Program

$$\max 260 S + 245 LX - 385,000 \quad (\text{Net Profit})$$

subject to:

$$\text{(S assembly)} \quad S \leq 1900$$

$$\text{(LX assembly)} \quad LX \leq 1400$$

$$\text{(Stamping)} \quad 0.3 S + 0.3 LX \leq 800$$

$$\text{(Forming)} \quad 0.25 S + 0.5 LX \leq 800$$

$$\text{(Non-negativity)} \quad S, LX \geq 0$$

- ▶ Optimal solution:  $S = 1900$ ,  $LX = 650$ , Net Profit = \$268,250.

- The answer report gives the slack (i.e., unused capacity) for each constraint. A constraint is *binding*, or *tight*, if the slack is zero (i.e., all of the capacity is used).
- The results from the sensitivity and answer reports are summarized next.

$$\max 260 S + 245 LX - 385,000 \quad (\text{Net Profit})$$

subject to:

			Slack	Shadow Price
(S assembly)	S	$\leq 1900$	0	137.5
(LX assembly)		$LX \leq 1400$	750	0
(Stamping)	0.3 S + 0.3 LX	$\leq 800$	35	0
(Forming)	0.25 S + 0.5 LX	$\leq 800$	0	490
(S non-neg.)	S	$\geq 0$	1900	0
(LX non-neg.)		$LX \geq 0$	650	0

Optimal solution:  $S = 1900$ ,  $LX = 650$ , Net Profit = \$268,250.

- In general,

$$\text{Slack} > 0 \Rightarrow \text{Shadow Price} = 0$$

and

$$\text{Shadow Price} > 0 \Rightarrow \text{Slack} = 0$$

It is possible to have a shadow price equal to 0 and a slack equal to 0.

## Objective Coefficient Ranges

### Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$4	Production per month Model S	1900	0	260	1E+30	137.5
\$D\$4	Production per month Model LX	650	0	245	275	245

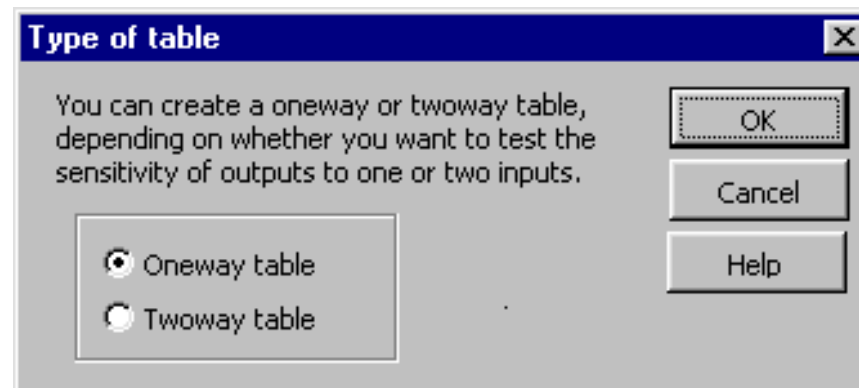
### Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$15	Model S assembly Used	1900	137.5	1900	233.33333	1500
\$E\$16	Model LX assembly Used	650	0	1400	1E+30	750
\$E\$17	Stamping (hours) Used	765	0	800	1E+30	35
\$E\$18	Forming (hours) Used	800	490	800	58.333333	325

- The Adjustable Cells section of the sensitivity report also contains objective coefficient ranges.
- For example, the optimal production plan will not change if the profit contribution of model *LX* increases by at most 275 or decreases by at most 245 from the current value of 245. (The optimal profit will change, but the optimal production plan remains at  $S = 1900$  and  $LX = 650$ .)
- Further, the optimal production plan will not change if the profit contribution of model *S* increases by any amount. Why? At a production level of  $S = 1900$ , Shelby is already producing as many model *S* shelves as possible.

## Using the SolverTable Add-in

- Suppose you would like to determine the optimal profit for different Model S assembly capacities ranging from 0 to 4000 units in increments of 100 units.
- SolverTable enables you to set up a number of optimization models by varying a cell (or cells) incrementally and, for each, it solves the problem and records the values in specified cells.
- *Using SolverTable:*
  - ▶ To load the SolverTable Add-in into Excel, download the files from the course web-site and follow the instructions in the solvertable.html file.
  - ▶ It is possible to create a Oneway table or a Twoway table, depending on how many cells you want to vary. Here we will do a Oneway table.
  - ▶ Go to *Data/SolverTable* and you will get the following dialog box:



- ▶ Click on “Oneway table” and “OK”.
- ▶ Then you will get the following dialog box:

## Using the SolverTable Add-in (continued)

**Parameters for oneway table** [X]

If you already ran a oneway SolverTable on this sheet, the previous settings are shown. Of course, you can enter new values if you like.

Input cell:

OK  
Cancel

Values of input to use for table

Base input values on following:

Minimum value:

Maximum value:

Increment:

Use the values below (separate with commas)

Input values:

Output cell(s):

Location of table:  (upper left cell of table)

Note: Be careful. The table will write over anything in its way! You might want to delete any old tables before creating any new ones.

## Using the SolverTable Add-in (continued)

- *Enter the following:*
  - ▶ *Input cell:* This is the cell that you want to change, so we specify the S Assembly Capacity cell (G15).
  - ▶ *Values of input to use for table:* Specify the range of values for the input cell, 0 for *Minimum Value*, 4000 for *Maximum Value* and 100 for *Increment*.
  - ▶ *Output cell(s):* Specify the cells whose value you want to record during the process (e.g., Optimal Profit at H5, and the optimal production quantities at C4:D4). Multiple ranges should be separated by a comma.
  - ▶ *Location of Table:* Locate the table in some blank part of your spreadsheet or in a new worksheet. (It may be safer to locate the output on the same sheet.)

## SolverTable (continued)

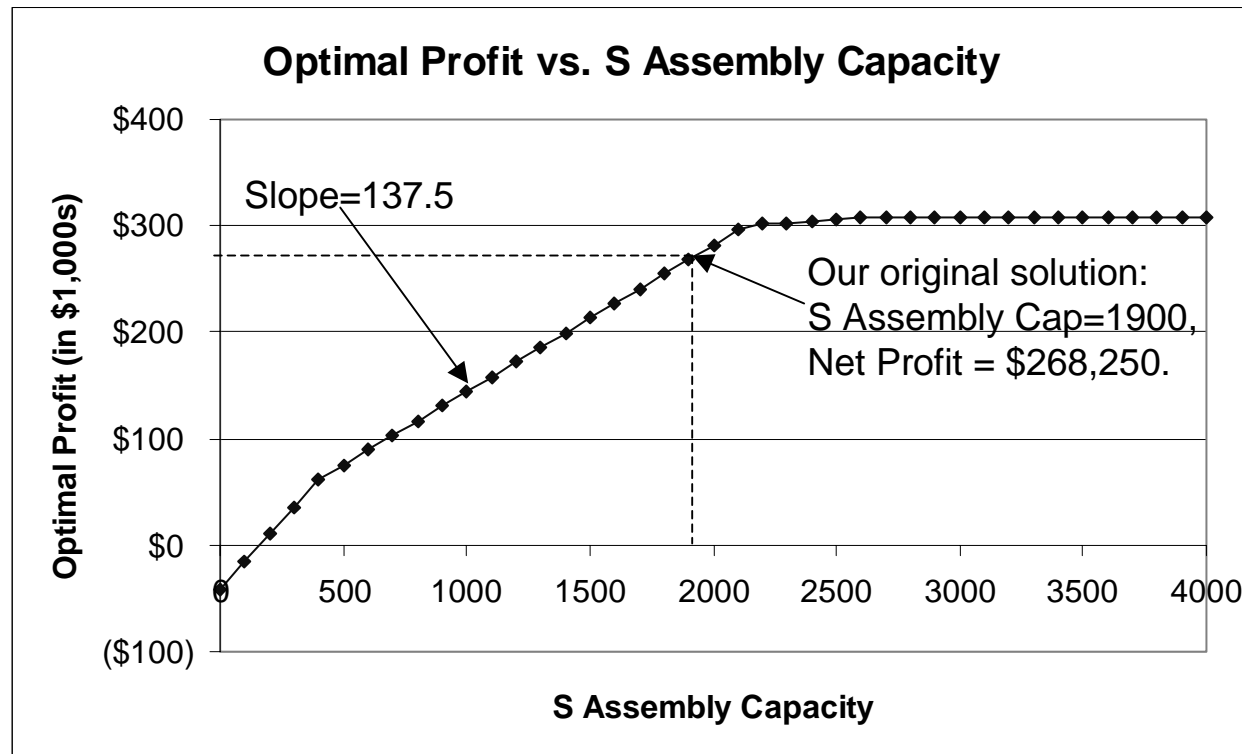
- After clicking “OK”, SolverTable will take some time to solve these problems. It will then produce a “table”, the top of which is shown here:

Different Values of S Assembly Capacity	Optimal Profit			Comments
	\$H\$5	\$C\$4	\$D\$4	
0	(\$42,000.00)	0	1400	
100	(\$16,000.00)	100	1400	
200	\$10,000.00	200	1400	
300	\$36,000.00	300	1400	
400	\$62,000.00	400	1400	
500	\$75,750.00	500	1350	
600	\$89,500.00	600	1300	

- The table lists the output for all the optimization problems.
- For each it records the input cell (Model S Assembly Capacity) and each of the output cells specified: Optimal Profit (\$H\$5) and the optimal production quantities of both Model S (\$C\$4) and Model LX (\$D\$4).
- SolverTable inserts comments (the red cell corners) at each value of Net Profit. These comments give information about the problem: for example, whether an optimal solution was found for that problem or whether the problem was infeasible.

## Optimal Objective Function versus Right-hand Side

- Using the output from the SolverTable we can make the following graph:



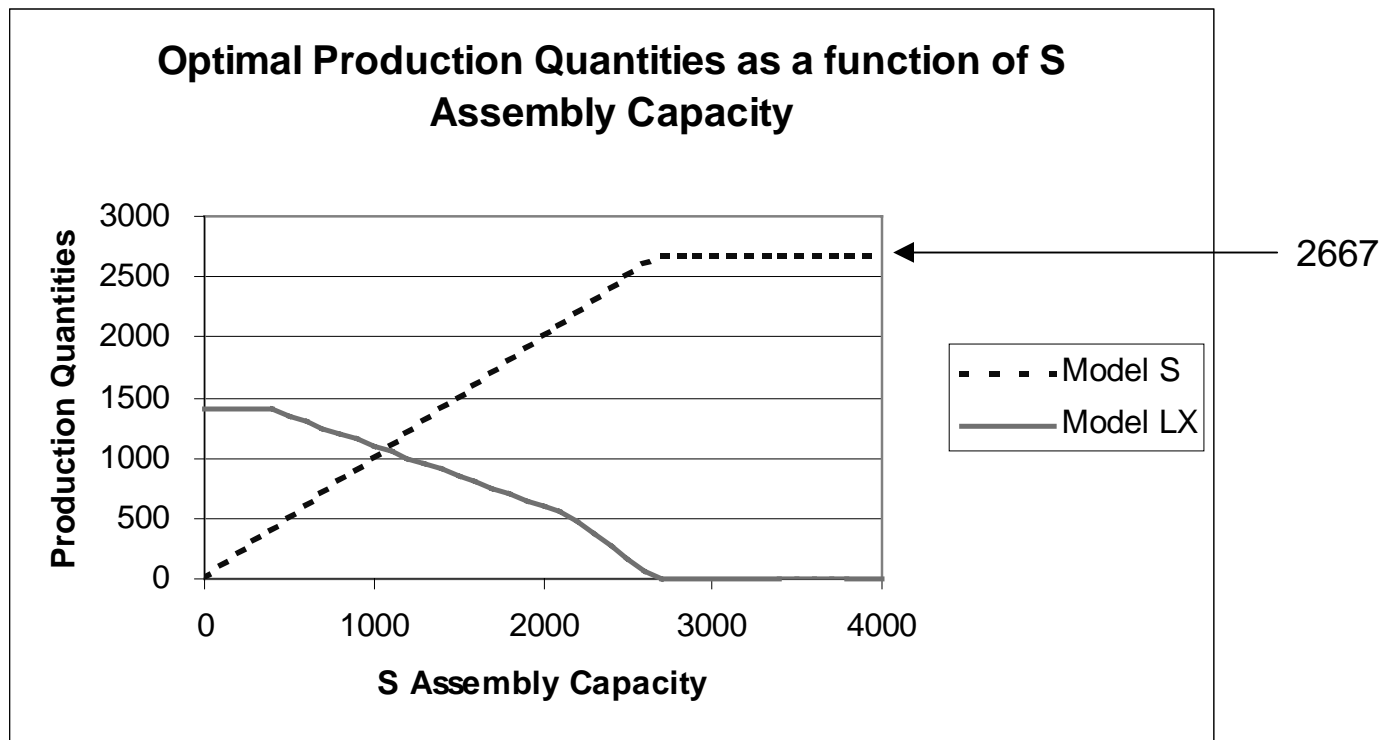
- This graph shows how the optimal profit varies as a function of the Model S assembly capacity.
- The slope of the graph is the shadow price of the Model S assembly capacity:

$$\text{Slope} = \frac{\text{Change in optimal profit}}{\text{Change in RHS}} = \text{Shadow Price}$$



## Optimal Production Quantities versus Right-hand Side

- We can also graph the optimal production quantities as a function of the right-hand side (S Assembly Capacity) as follows:



- As S Assembly capacity increases, more and more resources are allocated to that product. In fact, from the graph we can discern that Model S is always produced at capacity, as long as that capacity is less than or equal to the value 2667.

## The Petromor Bidding Problem

- Petromor is selling land with good oil-extraction potential.
- Oil companies present sealed offers (\$ per barrel) for the zones that they are interested in buying.
- No oil company can be awarded more than one zone as a result of the public offering.
- Petromor would like to maximize the revenue resulting from these sales.

**Table 1.** Bids (in \$ per Barrel)

	A	B	C	D	E	F
Zone 1	\$8.75	\$8.70	\$8.80	\$8.65	\$8.60	\$8.50
Zone 2	\$6.80	\$7.15	\$7.25	\$7.00	\$7.20	\$6.85
Zone 3	\$8.30	\$8.20	\$8.70	\$7.90	\$8.50	\$8.40
Zone 4	\$7.60	\$8.00	\$8.10	\$8.00	\$8.05	\$7.85

**Table 2.** Zone potential (in # of Barrels)

	Potential
Zone 1	205,000
Zone 2	240,000
Zone 3	215,000
Zone 4	225,000

What is the most profitable assignment of zones to the companies in this case?

## Petromor Bidding Formulation

- *Indices:* To index the zones, let  $i = 1, 2, 3, 4$ . To index the companies, let  $j = A, B, \dots, F$ .

- *Decision Variables:* Let

$$X_{ij} = \begin{cases} 1 & \text{if zone } i \text{ is assigned to company } j \\ 0 & \text{otherwise} \end{cases}$$

- *Objective Function:*

$$\begin{aligned} \max \quad & 205,000(8.75X_{1A} + 8.70X_{1B} + \dots + 8.50X_{1F}) \\ & + 240,000(6.80X_{2A} + 7.20X_{2B} + \dots + 6.85X_{2F}) \\ & + 215,000(8.30X_{3A} + 8.20X_{3B} + \dots + 8.40X_{3F}) \\ & + 225,000(7.60X_{4A} + 8.00X_{4B} + \dots + 7.85X_{4F}) \end{aligned}$$

- *Constraints:*

- ▶ Every zone must be assigned to some company
- ▶ Total number of companies assigned to each zone = 1

This leads to four constraints:

$$\begin{aligned} \text{(Zone 1)} \quad & X_{1A} + X_{1B} + X_{1C} + X_{1D} + X_{1E} + X_{1F} = 1 \\ \text{(Zone 2)} \quad & X_{2A} + X_{2B} + X_{2C} + X_{2D} + X_{2E} + X_{2F} = 1 \\ \text{(Zone 3)} \quad & X_{3A} + X_{3B} + X_{3C} + X_{3D} + X_{3E} + X_{3F} = 1 \\ \text{(Zone 4)} \quad & X_{4A} + X_{4B} + X_{4C} + X_{4D} + X_{4E} + X_{4F} = 1 \end{aligned}$$

## Petromor Bidding Formulation (continued)

- *Constraints (continued):*
  - ▶ Every company can be assigned at most one zone
  - ▶ Total number of zones assigned to each company  $\leq 1$

This leads to six constraints:

$$\text{(Company A)} \quad X_{1A} + X_{2A} + X_{3A} + X_{4A} \leq 1$$

$$\text{(Company B)} \quad X_{1B} + X_{2B} + X_{3B} + X_{4B} \leq 1$$

$$\text{(Company C)} \quad X_{1C} + X_{2C} + X_{3C} + X_{4C} \leq 1$$

$$\text{(Company D)} \quad X_{1D} + X_{2D} + X_{3D} + X_{4D} \leq 1$$

$$\text{(Company E)} \quad X_{1E} + X_{2E} + X_{3E} + X_{4E} \leq 1$$

$$\text{(Company F)} \quad X_{1F} + X_{2F} + X_{3F} + X_{4F} \leq 1$$

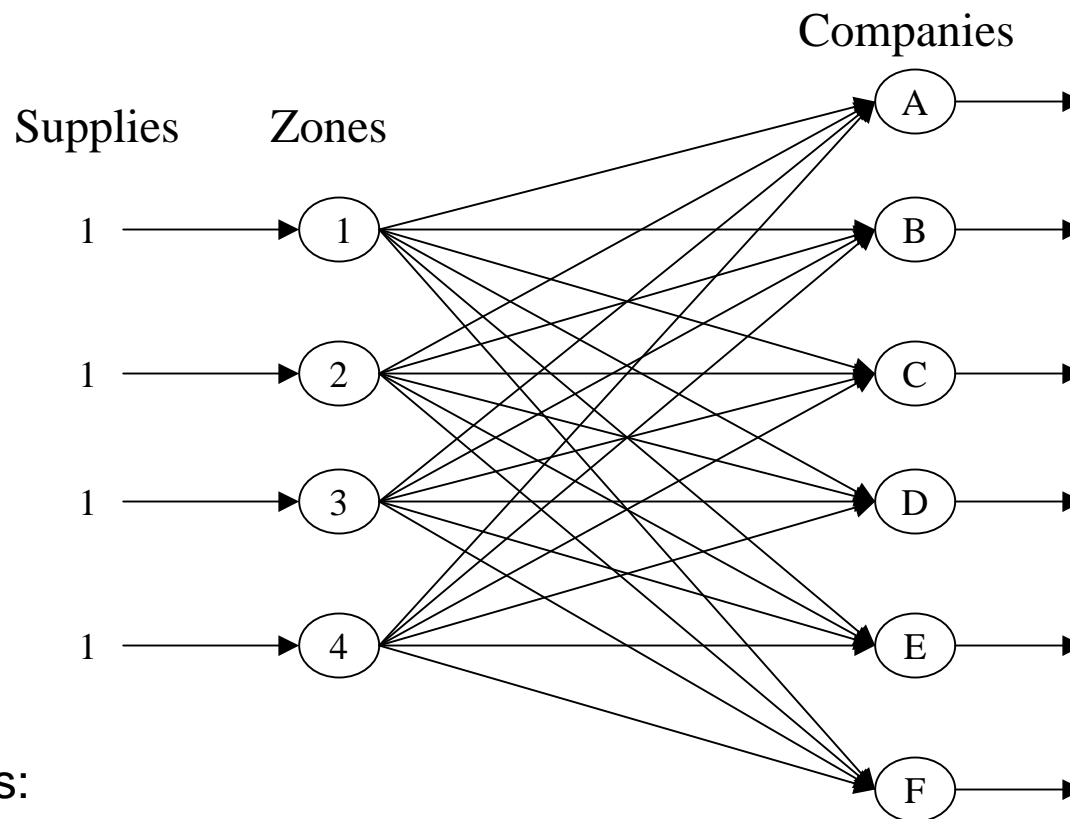
- ▶ Finally, the nonnegativity constraints:

$$X_{ij} \geq 0, \quad i=1, 2, 3, 4, \quad j=A, B, C, D, E, F.$$

- Should we add constraints restricting the decision variables to take on integer values only?

# Network Model

- It is *not* necessary to restrict the decision variables to take integer values. Integer values will occur automatically, since the formulation is a *network linear program*, that is, it can be drawn as a network with nodes and arcs, where some nodes have supplies or demands.



## Constraints:

- For every zone: Total bids out = 1
- For every company: Total bids in  $\leq 1$

## Assignment Models

- Since there are no transshipment nodes (i.e., each node has either positive supply or positive demand), and since the supply at each source is one, the model is called an *assignment model*. These models are frequently used for:
  - Assigning tasks to workers/machines
    - ▶ For scheduling operations
    - ▶ Classrooms, roommate assignments
  - Bidding for Awards and Contracts:
    - ▶ The New York City Department of Sanitation uses a similar model to assign contracts for garbage disposal.
    - ▶ The Bureau of Land Management of the Department of the Interior holds bimonthly simultaneous drawings enabling the public to acquire leases on large land parcels. A multibillion dollar industry of professional filing services assists investors in selecting parcels. One of these firms uses a similar model to assign clients to land-parcel applications.

# Bidding Problem Optimized Spreadsheet

Objective Function

=SUMPRODUCT(B20:G23, B13:G16)

A	A	B	C	D	E	F	G	H	I	J
1	PETROMOR.XLS		Petromor Oil Company							
2										
3	Revenue (in '000)						\$7,192.3			
4	Bids (in \$ per barrel)						Extraction Potential			
5		A	B	C	D	E	F			
6	Zone 1	\$8.75	\$8.70	\$8.80	\$8.65	\$8.60	\$8.50	205,000		
7	Zone 2	\$6.80	\$7.15	\$7.25	\$7.00	\$7.20	\$6.85	240,000		
8	Zone 3	\$8.30	\$8.20	\$8.70	\$7.90	\$8.50	\$8.40	215,000		
9	Zone 4	\$7.60	\$8.00	\$8.10	\$8.00	\$8.05	\$7.85	225,000		
10										
11	Bids per well (in thousands)									
12		A	B	C	D	E	F			
13	Zone 1	\$1,794	\$1,784	\$1,804	\$1,773	\$1,763	\$1,743			
14	Zone 2	\$1,632	\$1,716	\$1,740	\$1,680	\$1,728	\$1,644			
15	Zone 3	\$1,785	\$1,763	\$1,871	\$1,699	\$1,828	\$1,806			
16	Zone 4	\$1,710	\$1,800	\$1,823	\$1,800	\$1,811	\$1,766			
17										
18	Bids Assigned									
19		A	B	C	D	E	F	Total		
20	Zone 1	1	0	0	0	0	0	1		
21	Zone 2	0	0	0	0	1	0	1		
22	Zone 3	0	0	1	0	0	0	1		
23	Zone 4	0	1	0	0	0	0	1		
24	Total	1	1	1	0	1	0			

=G6\*\$I6/1000 and  
copied to B13:G16

H20:H23  
constrained  
to be =1

=SUM(F20:F23)

Decision variables B20:G23

B24:G24  
constrained  $\leq 1$

- Decision variables: Located in cells B20:G23.
- Objective function to be maximized is cell G3.
- Constraints are indicated in the spreadsheet.

## Bidding Problem: Solver Parameters

**Solver Parameters**

Set Target Cell:

Equal To:  Max  Min  Value of:

By Changing Cells:

Subject to the Constraints:

- 
- 

Buttons: Solve, Close, Options, Reset All, Help

- Remember “Assume linear model” and “Assume Non-Negative” are checked in the Options dialog box.



# Bidding Problem Sensitivity Report

## Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$20	Zone 1 A	1	0	1793.75	1E+30	10.25
\$C\$20	Zone 1 B	0	0	1783.5	10.25	2.75
\$D\$20	Zone 1 C	0	-3	1804	2.75	1E+30
\$E\$20	Zone 1 D	0	-10	1773.25	10.25	1E+30
\$F\$20	Zone 1 E	0	-32	1763	31.75	1E+30
\$G\$20	Zone 1 F	0	-41	1742.5	41	1E+30
\$B\$21	Zone 2 A	0	-95	1632	95	1E+30
\$C\$21	Zone 2 B	0	-1	1716	0.75	1E+30
\$D\$21	Zone 2 C	0	0	1740	31	0.75
\$E\$21	Zone 2 D	0	-37	1680	36.75	1E+30
\$F\$21	Zone 2 E	1	0	1728	0.75	0.75
\$G\$21	Zone 2 F	0	-73	1644	72.75	1E+30
\$B\$22	Zone 3 A	0	-73	1784.5	73	1E+30
\$C\$22	Zone 3 B	0	-84	1763	84.25	1E+30
\$D\$22	Zone 3 C	1	0	1870.5	1E+30	31
\$E\$22	Zone 3 D	0	-149	1698.5	148.75	1E+30
\$F\$22	Zone 3 E	0	-31	1827.5	31	1E+30
\$G\$22	Zone 3 F	0	-41	1806	41.25	1E+30
\$B\$23	Zone 4 A	0	-100	1710	100.25	1E+30
\$C\$23	Zone 4 B	1	0	1800	2.75	0
\$D\$23	Zone 4 C	0	-1	1822.5	0.75	1E+30
\$E\$23	Zone 4 D	0	0	1800	0	10.25
\$F\$23	Zone 4 E	0	0	1811.25	0.75	0.75
\$G\$23	Zone 4 F	0	-34	1766.25	33.75	1E+30

## Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$H\$20	Zone 1 Total	1	1,783	1	1	0
\$H\$21	Zone 2 Total	1	1,717	1	0	0
\$H\$22	Zone 3 Total	1	1,847	1	0	0
\$H\$23	Zone 4 Total	1	1,800	1	1	0
\$B\$24	Total A	1	10.25	1	0	1
\$C\$24	Total B	1	0	1	0	1
\$D\$24	Total C	1	23.25	1	0	0
\$E\$24	Total D	0	0	1	1E+30	1
\$F\$24	Total E	1	11.25	1	0	0
\$G\$24	Total F	0	0	1	1E+30	1

## Petromor Bidding Optimal Solution

	Zone 1	Zone 2	Zone 3	Zone 4
Company Assigned:	A	E	C	B

Total revenue from the sales: \$7,192.3 thousand.

Shadow prices and RHS ranges for flow-balance constraints (for each bidder):

Company	Shadow Price	Allowable Increase	Allowable Decrease
A	10.25	0	1
B	0	0	1
C	23.25	0	0
D	0	Infinity	1
E	11.25	0	0
F	0	Infinity	1

(Extra decimal places in the shadow prices are obtained by changing the numeric format of the Excel sensitivity report.)

## Interpretation of the Sensitivity Report I

- Company D is a fake company created by the owners of Company A, so as to circumvent the restriction that no more than one zone can be assigned to a company. Company D should have been eliminated from the bid.
- *Would the result of the optimization have been different?*
- No, because Company D was not assigned any zones. This means that the shadow price associated with the constraint limiting the number of bids assigned to Company D is zero, and hence, any changes in the RHS will not affect the optimal solution.

## Interpretation of the Sensitivity Report II

- After the envelopes with all the bids have been opened, all the bidding companies can find out what the other companies offered for the different zones. Mr. Vaco overheard the following statement from a senior analyst at company A: “Our offer was too high; we could have lowered it by almost \$0.10 a barrel, and still have been awarded Zone 1.”
- *Is it true that Company A could have lowered their bid for Zone 1 by \$0.10 and still have won the bidding?*
- From the sensitivity report, we can see that the objective function coefficient for Zone 1, Company A, could have been decreased by \$10,250 without affecting the result of the optimization. This means that Company A could have decreased their bid by at most \$0.05 per barrel ( $= \$10,250/205,000$ ) and still have won the bid. A decrease of \$0.10 per barrel is outside the range, so we would have to reoptimize to get the correct solution. This new solution does not assign Zone 1 to Company A.

## Interpretation of the Sensitivity Report III

- *What would happen if Company A decided to pull out from the bid?*
- We can answer this question by looking at the shadow price associated with Company A. If we do not assign any zones to Company A then the revenue would go down by \$10,250 (the RHS goes from 1 to 0, and the decrease is within the allowable decrease of 1).
- *What is the “hidden cost” of the policy that each company can be assigned at most one zone?*
- If each company can be assigned any number of zones, we need to delete the six company constraints “Total bids awarded  $\leq 1$ ” (i.e., the constraints on cells B24:G24 should be deleted). Since this question involves a change to six constraints, we need to reoptimize the model.
- The optimal revenue increases by \$44,750 to \$7,237,000. That is, the hidden cost of the policy that each company can be assigned to at most one zone is \$44,750.

## Summary

- Understand the optimizer sensitivity report
  - ▶ Shadow (or dual) prices
  - ▶ right hand side ranges
  - ▶ Objective coefficient ranges
- Petromor Assignment Model
  - ▶ Understanding the sensitivity report

### **For next class**

- Read Chapter 2.9 and 3.7 in the W & A text.