Lecture 2

- A Telephone Staffing Problem
- TransportCo Distribution Problem
- Shelby Shelving Case
- Summary and Preparation for next class

A Telephone Staffing Problem

- A market researcher is going to conduct a telephone survey to determine satisfaction levels with a popular household product.
- The survey must closely match their customer profile and deliver the required statistical accuracy. The survey will be conducted during one day.
- To achieve this, it is determined that they need to survey at least:
  - 240 wives
  - 180 husbands
  - 210 single adult males, and
  - 160 single adult females.
- The market researcher must hire temporary workers to work for one day. These workers make the phone calls and conduct the interviews. She has the option of hiring daytime workers, who work 8 hours (from 9am-5pm), or evening workers, who can work 3 hours (from 6pm-9pm).
- A daytime worker gets paid $10 per hour, while an evening worker gets paid $15 per hour.
- The market researcher wants to minimize the total cost of the survey.

A Telephone Staffing Problem (continued)

- Several different outcomes are possible when a telephone call is made to a home, and the probabilities differ depending on whether the call is made during the day or in the evening.
- The table below lists the results that can be expected:

Person Responding	Percentage of Daytime Calls	Percentage of Evening Calls
Wife	15	20
Husband	10	30
Single Male	10	15
Single Female	10	20
No Answer	55	15

- For example, 15% of all daytime calls are answered by a wife, and 15% of all evening calls are answered by a single male.
- A daytime caller can make 12 calls per hour, while an evening caller can make 10 calls per hour.
- Because of limited space, at most 20 people can work in any one shift (day or evening).
- Formulate the problem of minimizing cost as a linear program.

A Telephone Staffing Problem: Overview

- What needs to be decided?

  The number of workers to hire in each shift (day and evening).
- What is the objective?

  Minimize the cost.
- What are the constraints?

  There are minimum requirements for each category (wife, husband, single male and single female). There is a limit on the number of people working during each shift. There are non-negativity constraints.
- The Telephone Staffing Problem optimization model in general terms:

  min Total Cost

  subject to
  - Meet minimum requirements in each customer category
  - At most 20 workers per shift
  - Non-negative number of workers hired

A Telephone Staffing Problem: Model

- *Decision Variables*: Let
  - D = # of daytime workers to hire,
  - E = # of evening workers to hire,
- *Objective Function*: With the above decision variables, the total cost is
  - (\$10x8) D + (\$15x3) E = 80 D + 45 E
- Constraints:
  - Minimum Requirements in each customer category
    - (Wives) (0.15x12x8) D + (0.20x3x10) E $\geq$ 240
      - or 14.4 D + 6 E $\geq$ 240
    - (Husbands) (0.10x12x8) D + (0.30x3x10) E $\geq$ 180
      - or 9.6 D + 9 E $\geq$ 180

A Telephone Staffing Problem: Model

- Constraints (cont):
  - Minimum Requirements in each customer category
    - (Single Adult Mal.) $(0.10 \times 8 \times 12)\ D + (0.15 \times 3 \times 10)\ E \geq 210$
      - or $9.6\ D + 4.5\ E \geq 210$
    - (Single Adult Fem.) $(0.10 \times 8 \times 12)\ D + (0.20 \times 3 \times 10)\ E \geq 160$
      - or $9.6\ D + 6\ E \geq 160$
  - Limit on number of workers hired per shift
    - $D \leq 20$
    - $E \leq 20$
  - Non-negativity
    - $D \geq 0$, $E \geq 0$.

A Telephone Staffing Problem
Linear Programming Model

min $80D + 45E$

subject to:

(Wives)	$14.4D + 6E \geq 240$
(Husbands)	$9.6D + 9E \geq 180$
(Single Adult Males)	$9.6D + 4.5E \geq 210$
(Single Adult Females)	$9.6D + 6E \geq 160$
(Limit on Day Workers)	$D \leq 20$
(Limit on Eve. Workers)	$E \leq 20$
(Non-negativity)	$D \geq 0, E \geq 0$

A Telephone Staffing Problem
Optimized Spreadsheet

	A	B	C	D	E	F	G
1	STAFFING.XLS	**Telephone Staffing Problem**					
2							
3		**Day**	**Evening**				
4	Shift	9am-5pm	6-10pm				
5	Hours per shift	8	3		Total Cost		
6	Calls per hour	12	10		**$ 1,780**		
7	Cost per hour	$ 10	$ 15				
8	Cost per worker	$ 80	$ 45				
9							
10	Number of workers to hire	**20**	**4**				
11		<=	<=				
12	Limit	20	20				
13							
14	Number of Calls	1920	120				
15						Minimum	
16	**Expected Results**	**Day**	**Evening**	Total		Requirement	
17	Wives	15%	20%	312.0	>=	240	
18	Husbands	10%	30%	228.0	>=	180	
19	Single Adult Males	10%	15%	210.0	>=	210	
20	Single Adult Females	10%	20%	216.0	>=	160	
21	No Answer	55%	15%	1,074.0			
22							

`=SUMPRODUCT(B8:C8,B10:C10)` (Total Cost, E6)

`=C7*C5` (C8)

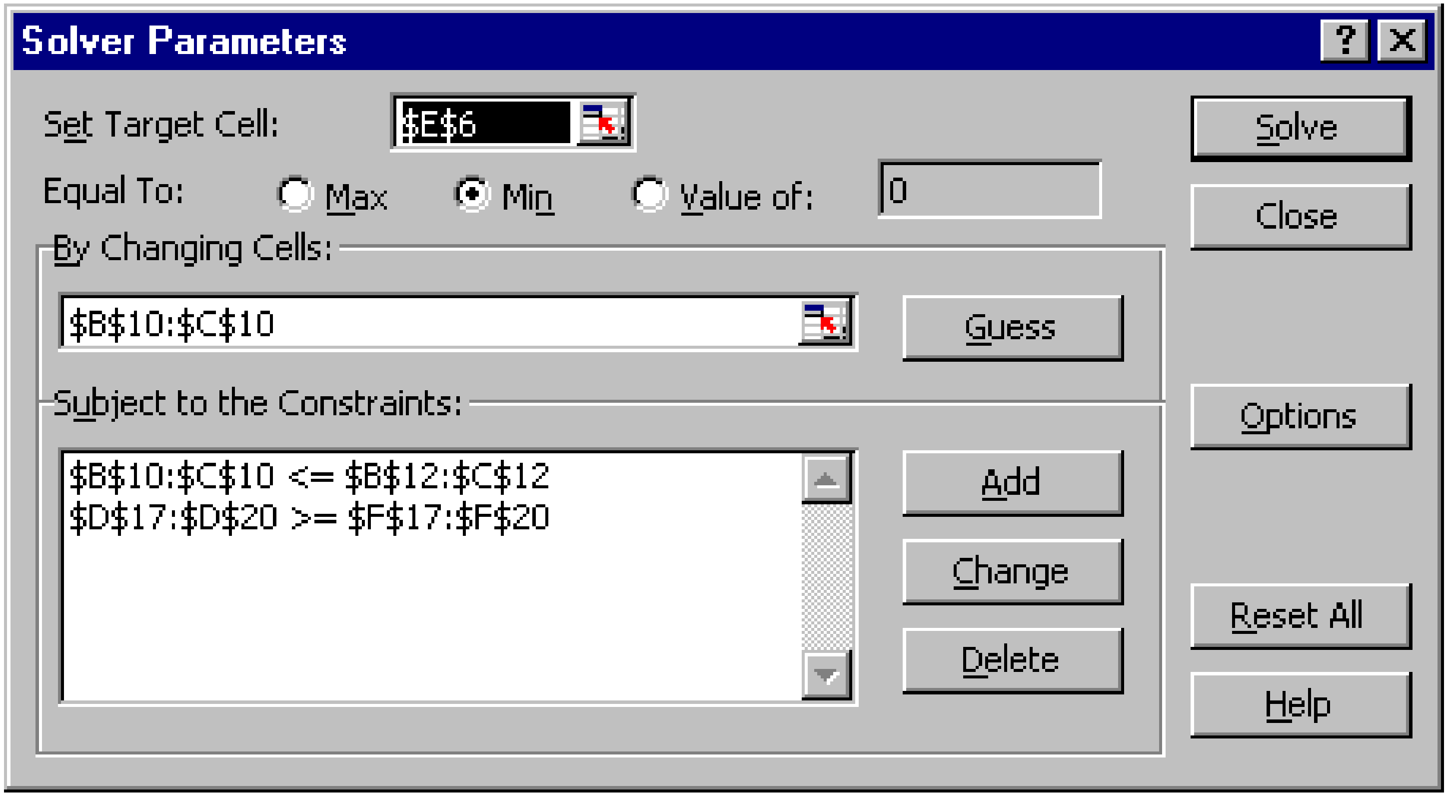
Decision Variables (B10:C10)

`=C6*C5*C10` (C14)

`=SUMPRODUCT($B$14:$C$14,B21:C21)and copied to D17:D21`

`=IF(D20>=F20-0.00001,">=","Not >=")`

A Telephone Staffing Problem: Solver Parameters



Solver Parameters for the Telephone Staffing Problem

A Telephone Staffing Problem: Solution Summary

- The optimal solution specifies to hire 20 daytime workers and only 4 evening workers.
- The total cost is $1,780.
- This strategy expects to survey 312 wives, 228 husbands, 210 single adult males and 216 single adult females.
- At most 20 workers are hired in any one shift.

Additional Comments

- Note that the model uses averages (expected values) and therefore the number of people contacted may actually vary from these averages.
- What happens if the solution specifies hiring fractional numbers of people?

TransportCo Distribution Problem

- TransportCo supplies goods to four customers, each requiring the following amounts:

	Demand Requirement (in units)
Nashville	25
Cleveland	35
Omaha	40
St. Louis	20

- The company has three warehouses with the following supplies available:

	Supply Available (in units)
Dallas	50
Atlanta	20
Pittsburgh	50

TransportCo Distribution Problem (cont.)

- The costs of shipping one unit from each warehouse to each customer are given by the following table:

		To			
		Nashville	Cleveland	Omaha	St. Louis
From	Dallas	$30	$55	$35	$35
From	Atlanta	$10	$35	$50	$25
From	Pittsburgh	$35	$15	$40	$30

- Construct a decision model to determine the minimum cost of supplying the customers.

TransportCo Distribution Problem Overview

- What needs to be decided?

  A distribution plan, i.e., the number of units shipped from each warehouse to each customer.

- What is the objective?

  Minimize the total shipping cost. This total shipping cost must be calculated from the decision variables.

- What are the constraints?

  Each customer must get the number of units they requested (and paid for). There are supply constraints at each warehouse.

- TransportCo optimization model in general terms:

  min Total Shipping Cost

  subject to

  - Demand requirement constraints
  - Warehouse supply constraints
  - Non-negative shipping quantities

TransportCo Distribution Model

- *Index:* Let D=Dallas, A=Atlanta, P=Pittsburgh, N=Nashville, C=Cleveland, O=Omaha and S=St. Louis.

- *Decision Variables*: Let

  X_{DN} = # of units sent from D=Dallas to N=Nashville,

  X_{DC} = # of units sent from D=Dallas to C=Cleveland,

  …..

  X_{PS} = # of units sent from P=Pittsburgh to S=St. Louis.

- *Objective Function*:

  With the decision variables we defined, the total shipping cost is:

$$30 X_{DN} + 55 X_{DC} + 35 X_{DO} + 35 X_{DS} + 10 X_{AN} + 35 X_{AC}$$
$$+ 50 X_{AO} + 25 X_{AS} + 35 X_{PN} + 15 X_{PC} + 40 X_{PO} + 30 X_{PS}$$

Demand and Supply Constraints

- *Demand Constraints*: In order to meet demand requirements at each customer, we need the following constraints:
  - For Nashville: $X_{DN} + X_{AN} + X_{PN} = 25$
  - For Cleveland: $X_{DC} + X_{AC} + X_{PC} = 35$
  - For Omaha: $X_{DO} + X_{AO} + X_{PO} = 40$
  - For St. Louis: $X_{DS} + X_{AS} + X_{PS} = 20$

- *Supply Constraints:* In order to make sure not to exceed the supply at the warehouses, we need the following constraints:
  - For Dallas: $X_{DN} + X_{DC} + X_{DO} + X_{DS} \leq 50$
  - For Atlanta: $X_{AN} + X_{AC} + X_{AO} + X_{AS} \leq 20$
  - For Pittsburgh: $X_{PN} + X_{PC} + X_{PO} + X_{PS} \leq 50$

TransportCo Linear Programming Model

min $30 X_{DN} + 55 X_{DC} + 35 X_{DO} + 35 X_{DS} + 10 X_{AN} + 35 X_{AC}$
$+ 50 X_{AO} + 25 X_{AS} + 35 X_{PN} + 15 X_{PC} + 40 X_{PO} + 30 X_{PS}$

subject to:

(Demand Constraints)

(Nashville) $X_{DN} + X_{AN} + X_{PN} = 25$

(Cleveland) $X_{DC} + X_{AC} + X_{PC} = 35$

(Omaha) $X_{DO} + X_{AO} + X_{PO} = 40$

(St. Louis) $X_{DS} + X_{AS} + X_{PS} = 20$

(Supply Constraints)

(Dallas) $X_{DN} + X_{DC} + X_{DO} + X_{DS} \leq 50$

(Atlanta) $X_{AN} + X_{AC} + X_{AO} + X_{AS} \leq 20$

(Pittsburgh) $X_{PN} + X_{PC} + X_{PO} + X_{PS} \leq 50$

Non-negativity: All variables ≥ 0

TransportCo Optimized Spreadsheet

Objective Function=`SUMPRODUCT(B7:E9,B13:E15)`

	A	B	C	D	E	F	G	H
1	TRANS.XLS		**TransportCo Distribution Problem**					
2								
3	Total Shipping Cost=		**$ 2,900**					
4								
5	Shipping Costs (per unit)							
6		Nashville	Cleveland	Omaha	St. Louis			
7	Dallas	$30	$55	$35	$35			
8	Atlanta	$10	$35	$50	$25			
9	Pittsburgh	$35	$15	$40	$30			
10								
11	Shipping Quantities (in units)					Total		
12		Nashville	Cleveland	Omaha	St. Louis	Shipped From		Supplies
13	Dallas	**5**	**0**	**40**	**5**	50	<=	50
14	Atlanta	**20**	**0**	**0**	**0**	20	<=	20
15	Pittsburgh	**0**	**35**	**0**	**15**	50	<=	50
16	Total Shipped to	25	35	40	20			
17		=	=	=	=			
18	Requirements	25	35	40	20			

Decision Variables

`=SUM(B13:B15)`

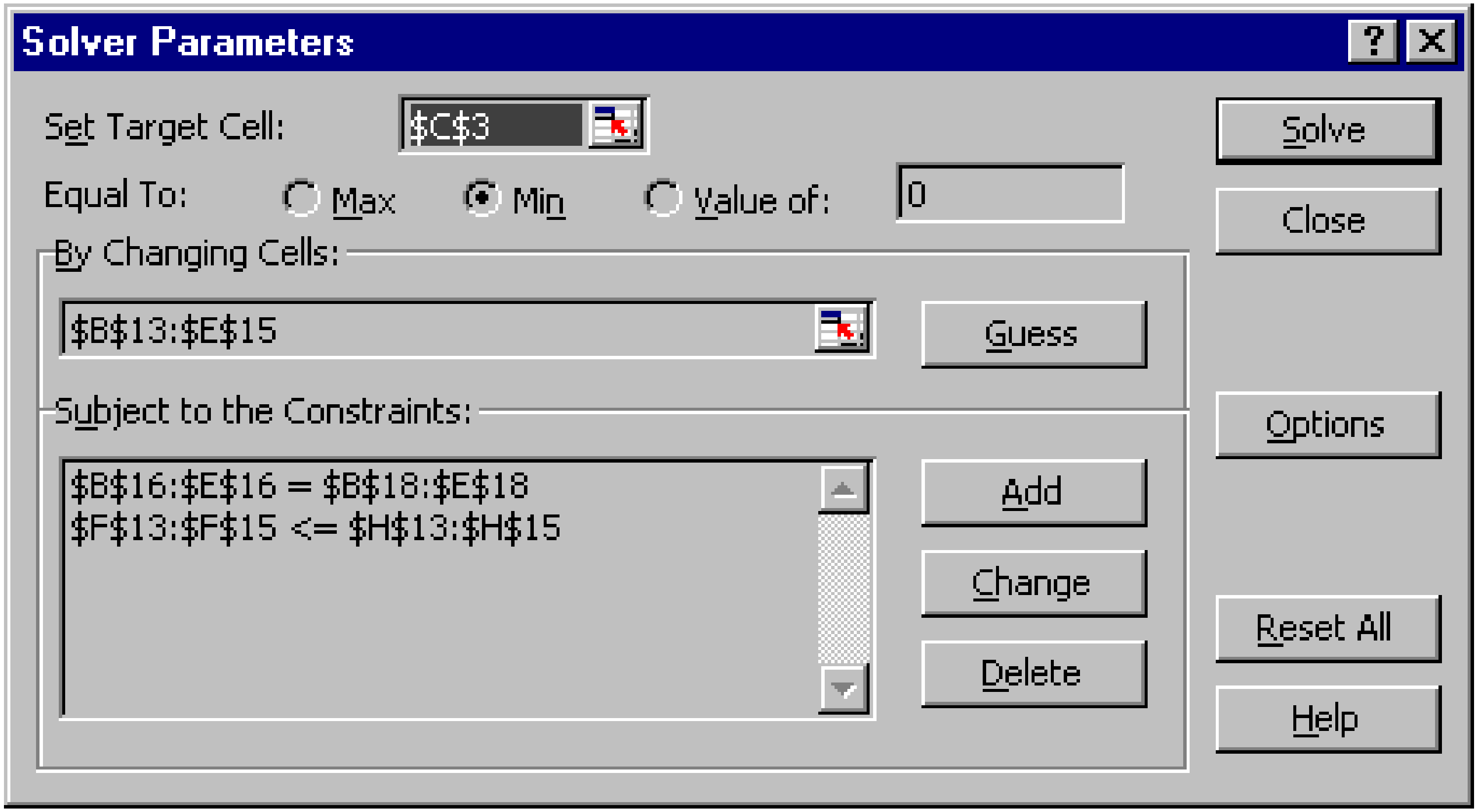
`=SUM(B13:E13)`

`=IF(ABS(B16-B18)<0.00001,"=","Not =")`

`=IF(F15<=H15+0.00001,"<=","Not <=")`

- The optimal solution has a total cost of $2,900.

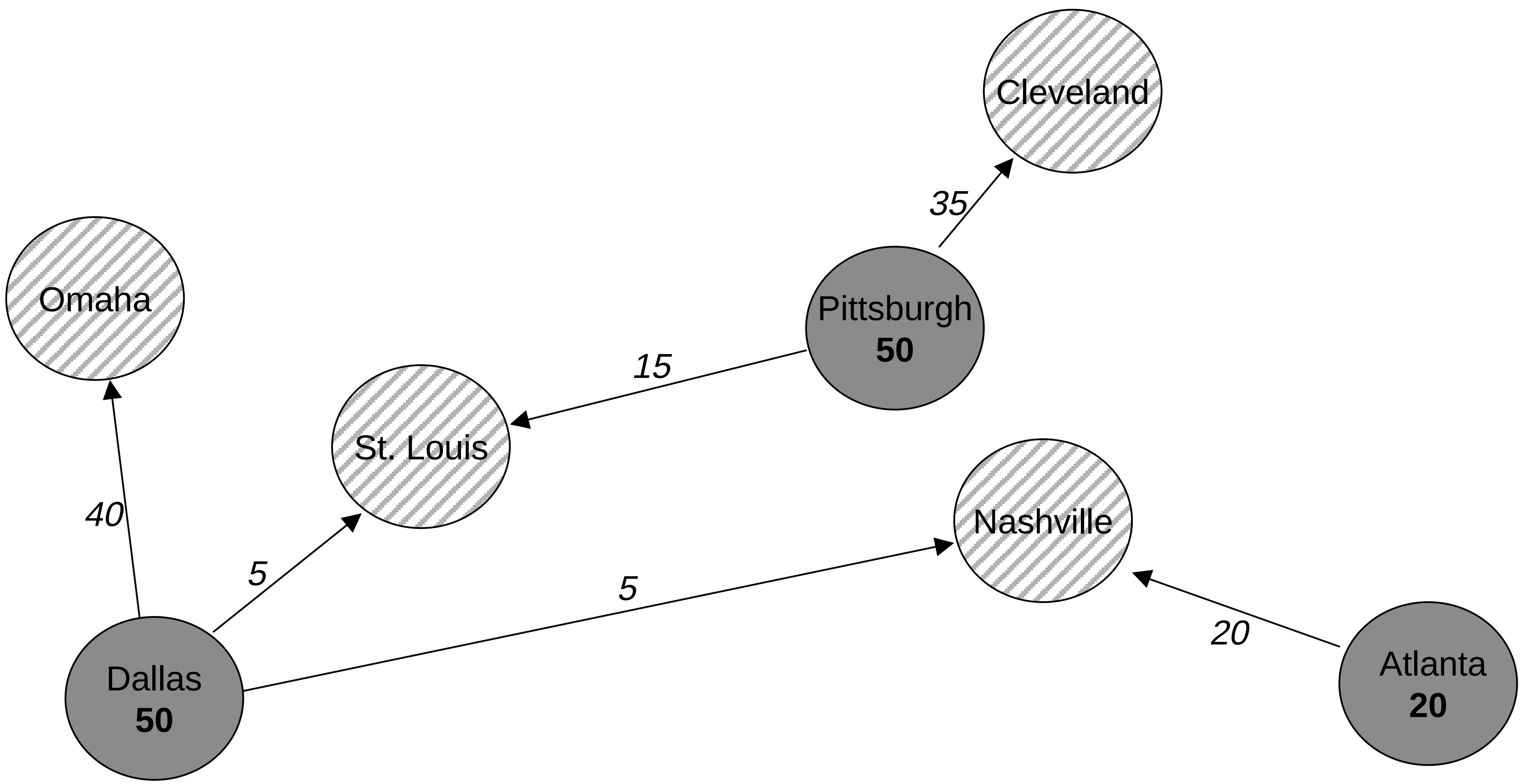
TransportCo Solver Parameters



The *Solver Parameters* dialog box with constraints added.

TransportCo Solution Summary

- The optimal solution has total cost $2,900.
- The optimal distribution plan is as follows:



Shelby Shelving Decision Model

- *Decision Variables*:

  Let S = # of Model S shelves to produce, and

  LX = # of Model LX shelves to produce.

- To specify the objective function, we need to be able to compute net profit for any production plan (S, LX). Case information:

	S	LX
Selling Price	1800	2100
Standard cost	1839	2045
Profit contribution	-39	55

$$\Rightarrow \text{ Net Profit} = -39\,S + 55\,LX \qquad (1)$$

So for the current production plan of S = 400 and LX = 1400, we get Net profit = $61,400.

- Is equation (1) correct?

- Equation (1) is not correct (although it does give the correct net profit for the *current* production plan). Why? Because the standard costs are based on the current production plan and they do not correctly account for the fixed costs for different production plans.

- For example, what is the net profit for the production plan $S = LX = 0$? Since

  Net Profit = Revenue - Variable cost - Fixed cost

  and Fixed cost = 385,000, the Net profit is -$385,000. But equation (1) incorrectly gives

  Net profit = $-39\ S + 55\ LX = 0$

  To derive a correct formula for net profit, we must separate the fixed and variable costs.

  Profit Contribution Calculation

	Model *S*	Model *LX*
a) Selling price	1800	2100
b) Direct materials	1000	1200
c) Direct labor	175	210
d) Variable overhead	365	445
e) Profit contribution (e = a-b-c-d)	260	245

- The correct objective function is

  Net profit = $260\ S + 245\ LX - 385{,}000$ (2)

Shelby Shelving LP

- *Decision Variables:*

  Let S = # of Model S shelves to produce, and

  LX = # of Model LX shelves to produce.

- Shelby Shelving Linear Program

$$\max \ 260\,S + 245\,LX - 385{,}000 \qquad \text{(Net Profit)}$$

subject to:

(S assembly)	$S \le 1900$
(LX assembly)	$LX \le 1400$
(Stamping)	$0.3\,S + 0.3\,LX \le 800$
(Forming)	$0.25\,S + 0.5\,LX \le 800$
(Nonnegativity)	$S,\ LX \ge 0$

Note: Net profit = Profit - Fixed cost, but since fixed costs are a constant in the objective function, maximizing Profit or Net Profit will give the same optimal solution (although the objective function values will be different).

Spreadsheet Solution

Decision Variables

Objective Function
+H3-H4

A	A	B	C	D	E	F	G	H	I
1	SHELBY.XLS		**Shelby Shelving Company**						
2									
3			Model S	Model LX		Gross profit		653,250.00	
4	Production per month		1900	650		Fixed cost		385,000.00	
5	Variable profit contribution		$260	$245		Net profit		$268,250.00	
6									
7	Selling price		1800	2100					
8	Direct materials		1000	1200					
9	Direct labor		175	210					
10	Variable overhead		365	445					
11	Variable profit contribution		260	245					
12									
13					Total		Total		
14			Usage per unit		Used	Constraint	Available		
15	Model S assembly		1	0	1900	<=	1900		
16	Model LX assembly		0	1	650	<=	1400		
17	Stamping (hours)		0.3	0.3	765	<=	800		
18	Forming (hours)		0.25	0.5	800	<=	800		
19									

SUMPRODUCT($C$4:$D$4, C15:D15)

SUMPRODUCT(C4:D4,C5:D5)

Summary

- Examples of two formulations: a telephone staffing problem and a transportation/distribution problem.
- Lesson from Shelby Shelving: Be careful about fixed versus variable costs

For next class

- Try question a) of the case Petromor: The Morombian State Oil Company. (Prepare to discuss the case in class, but do not write up a formal solution.)
- Read Chapter 4.4 in the W&A text.
- Load the SolverTable add-in to Excel. The needed files are available at the course web-page, where there are also instructions on how to do this.
- *Optional reading*: "Graphical Analysis" in the readings book.