COLUMBIA BUSINESS SCHOOL

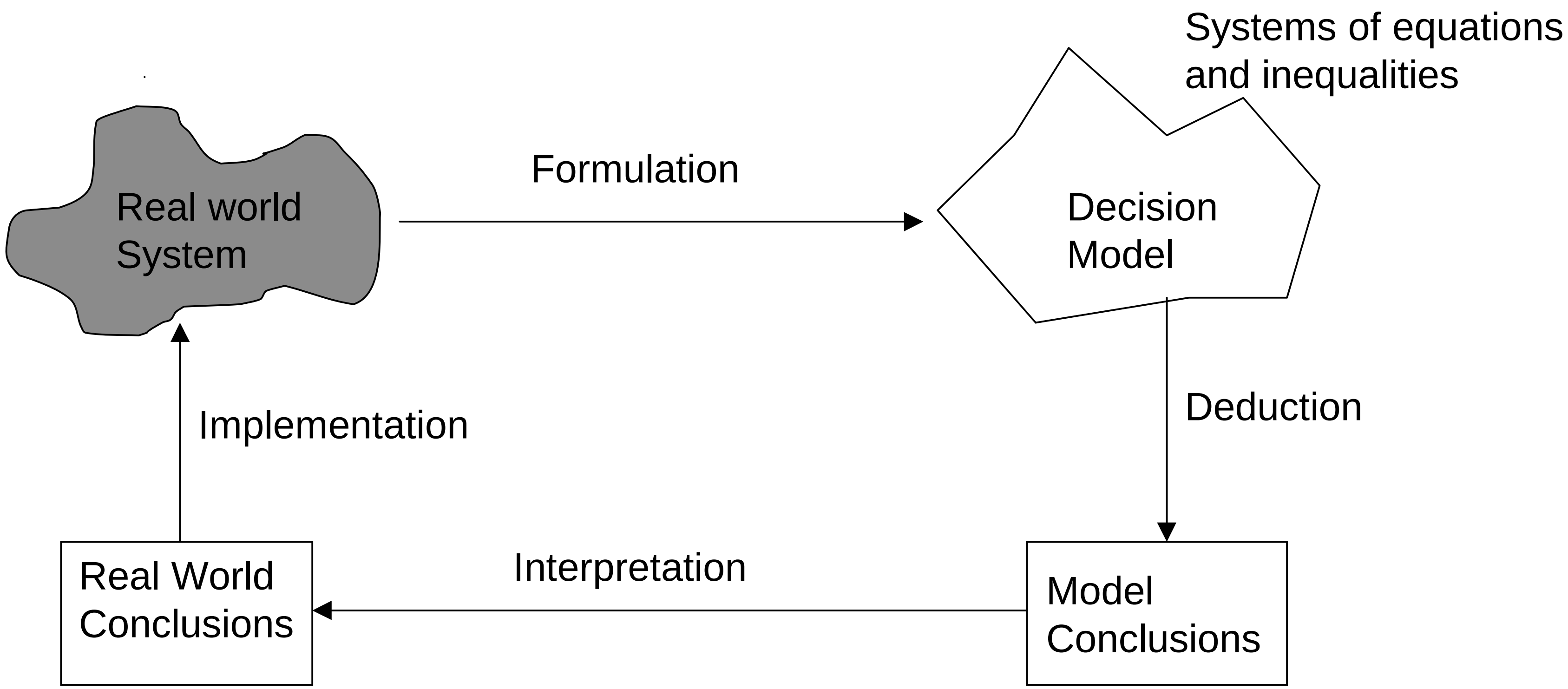
Lecture 1

- Syllabus Overview
- Introduction to Decision Models
- Bland Brewery Linear Programming Model
- Spreadsheet Optimization
- Summary and Preparation for next class

What is Decision Modeling?

- *Decision modeling* refers to the use of mathematical or scientific methods to determine an allocation of scarce resources that improves or optimizes the performance of a system.
- The terms *operations research* and *management science* are also used to refer to decision modeling.

Decision Modeling Process



Applications of Decision Models

A sample of *systems* to which decision models have been applied include:

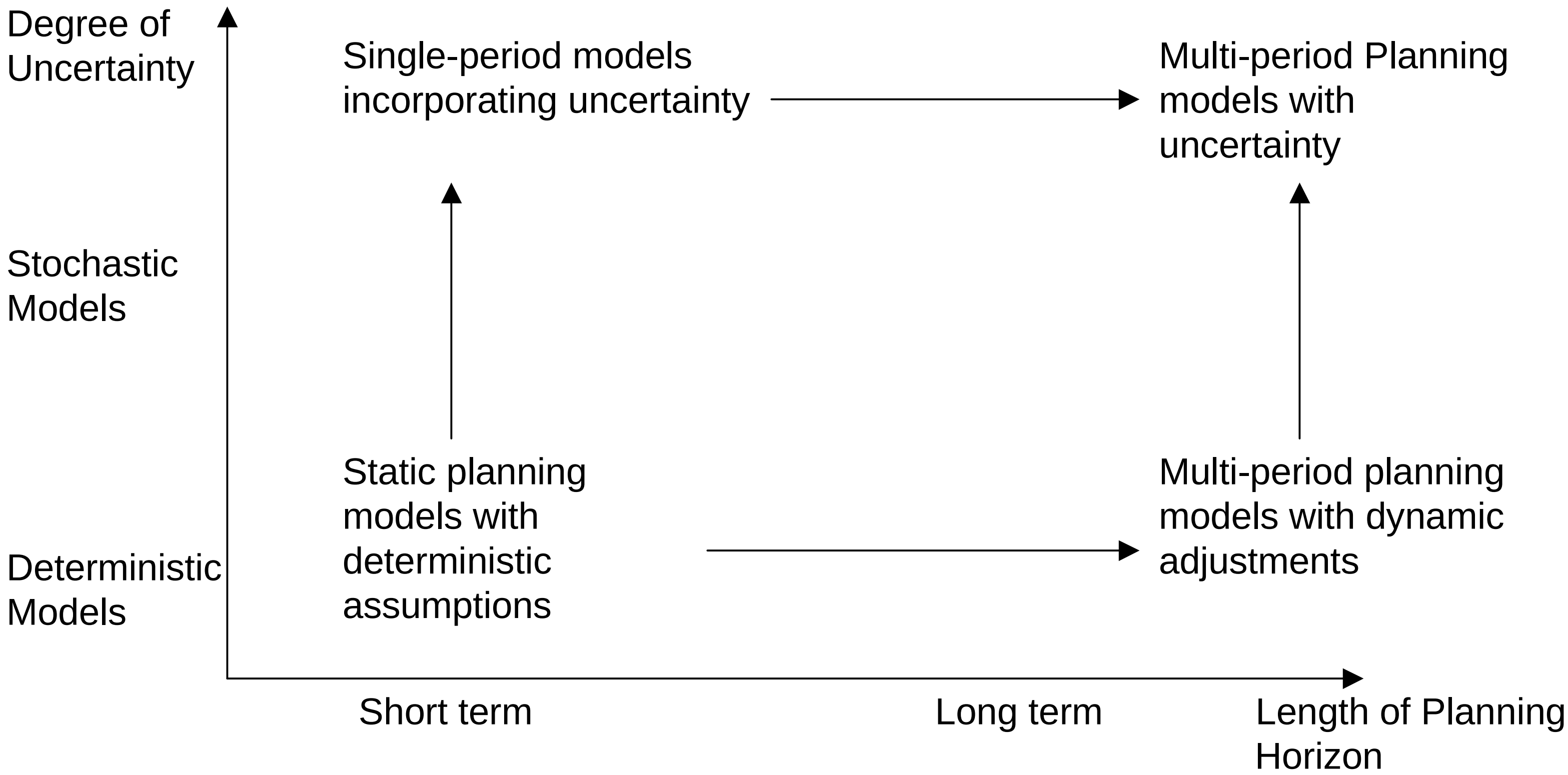
- Financial systems
  - Portfolio optimization, security pricing (e.g., options, mortgage-backed securities), cash-flow matching (e.g., pension planning and bond refunding)
  - Example: LibertyView Capital Management uses a spreadsheet optimization model developed by a 1995 Columbia MBA to hedge bond investments using stock and options
- Production systems
  - oil, steel, chemical, and many other industries
  - Example: Citgo uses linear programming to improve refining operations. Total benefit: approximately $70 million annually.

Applications of Decision Models (continued)

- Distribution systems
  - airlines, paper, school systems, and others
  - Example: Westvaco, a Fortune 200 paper company, uses linear programming to optimize its selection of motor carriers. The result: 3-6% savings on trucking costs of $15 million annually. This work was done by a 1992 Columbia MBA.
- Marketing systems
  - sales-force design, forecasting new-product sales, telecommunications strategies, brand choice, merchandising strategies
- Graduate school admissions
  - Example: The director of CBS admissions uses linear programming to aid in the admissions process.

References: The journal *Interfaces*, and the book *Excellence in Management Science Practice*, by Assad, Wasil, and Lilien, Prentice Hall, Englewood Cliffs, NJ (both are in the business school library).

Overview of Decision Models



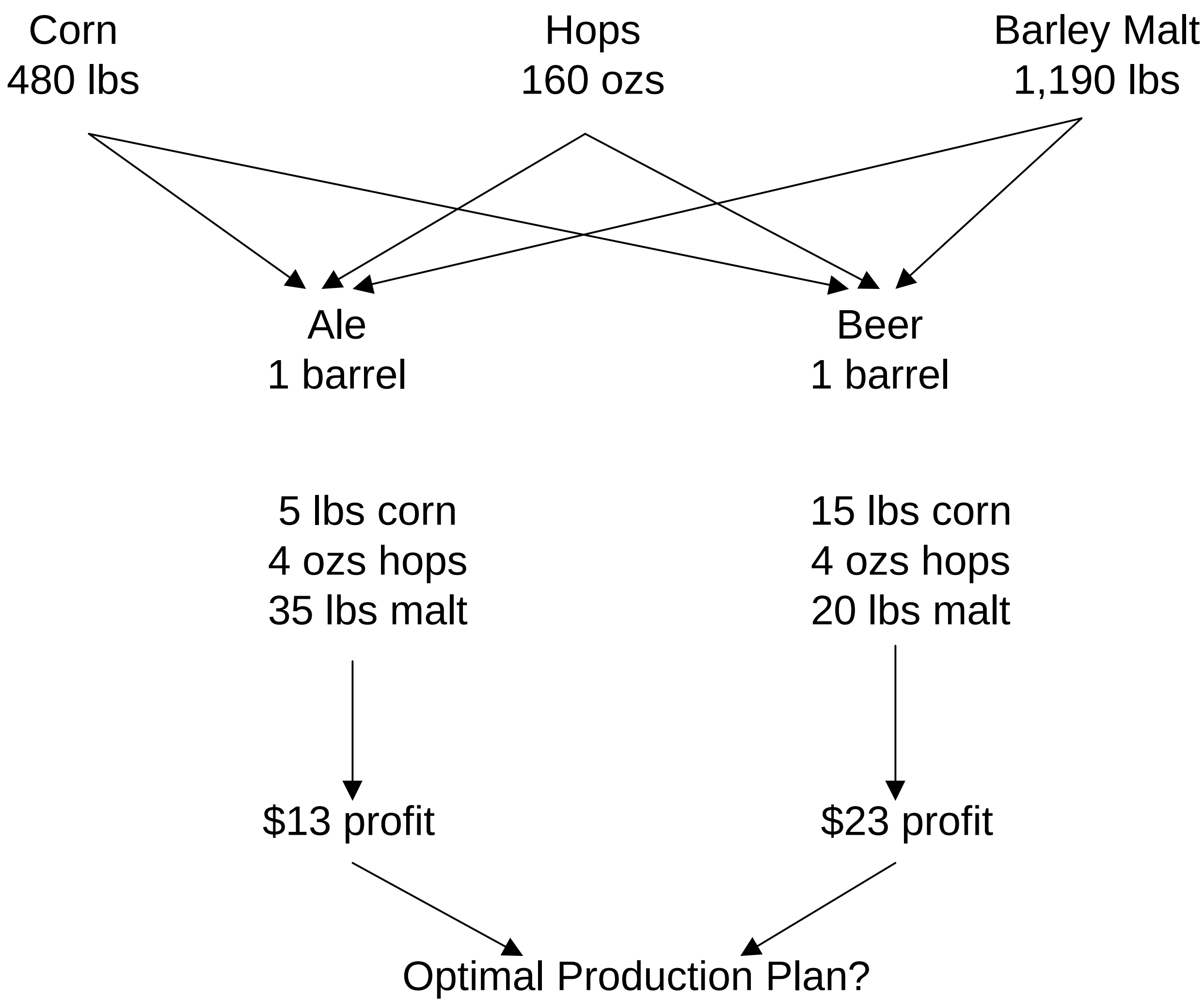
Main solution tools

- Optimization
  - Linear programming, Integer programming, Nonlinear programming
- Simulation

Bland Brewery Decision Problem

- Consider the situation of a small brewery whose ale and beer are always in demand but whose production is limited by certain raw materials that are in short supply. The scarce ingredients are corn, hops, and barley malt. The recipe for a barrel of ale calls for the ingredients in proportions different from those in the recipe for a barrel of beer. For instance, ale requires more malt per barrel than beer does. Furthermore, the brewer sells ale at a profit of $13 per barrel and beer at a profit of $23 per barrel. Subject to these conditions, how can the brewery maximize profit?

Bland Brewery Model



What if Bland decides to produce all ale? Then

- their corn supply limits production to at most 480/5 =96 barrels,
- their hops supply limits production to at most 160/4 =40 barrels, and
- their malt supply limits production to at most 1190/35 = 34 barrels.

Therefore, they can produce only 34 barrels of ale, which makes a profit of 34 $\times$ \$13 = \$442.

What if Bland decides to produce all beer? Then

- their corn supply limits production to at most 480/15 = 32 barrels,
- their hops supply limits production to at most 160/4 = 40 barrels, and
- their malt supply limits production to at most 1190/20 = 59.5 barrels.

Therefore, they can produce only 32 barrels of beer, which makes a profit of 32 $\times$ \$23 = \$736.

Is there a better production plan? One way to simplify the computations is to set up a spreadsheet.

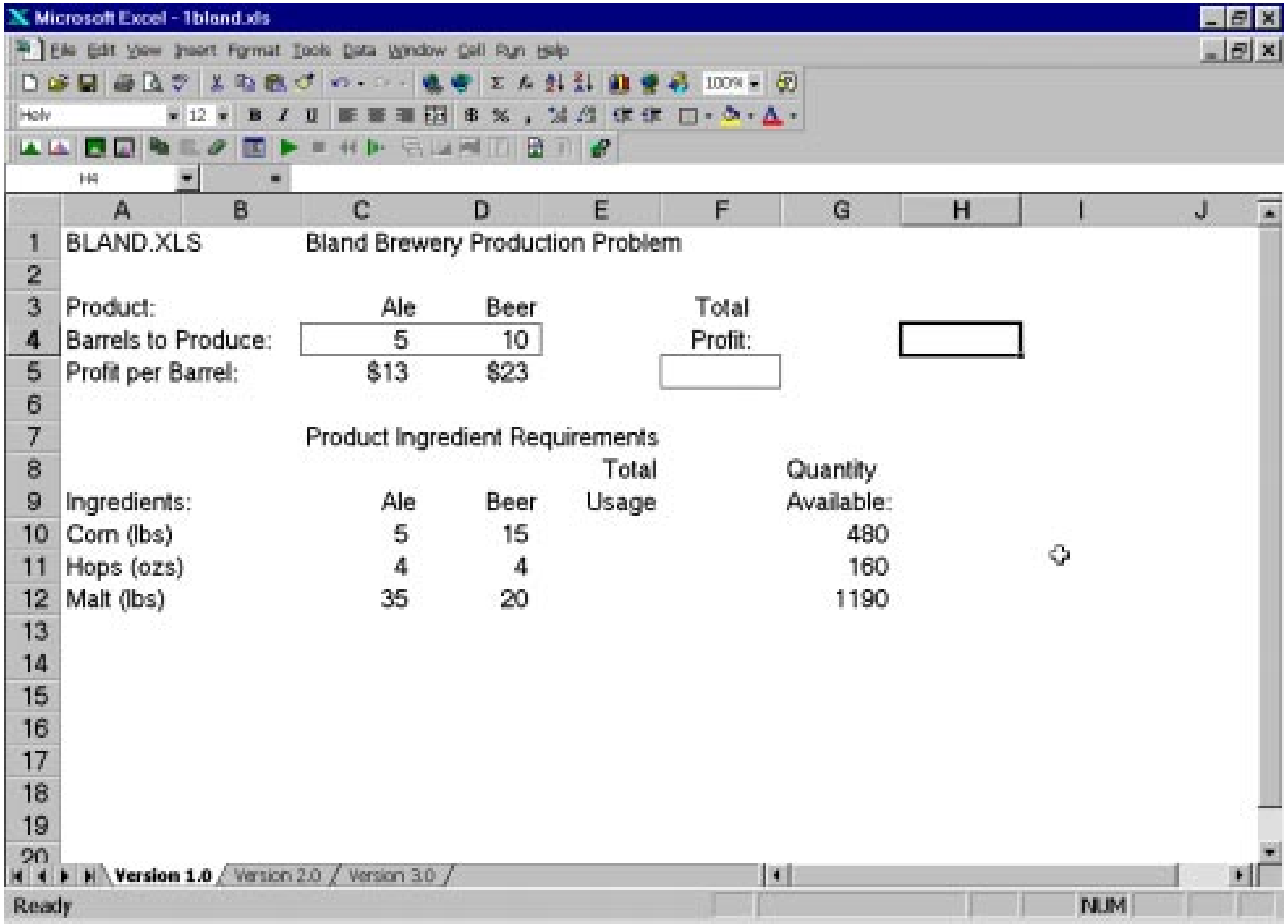
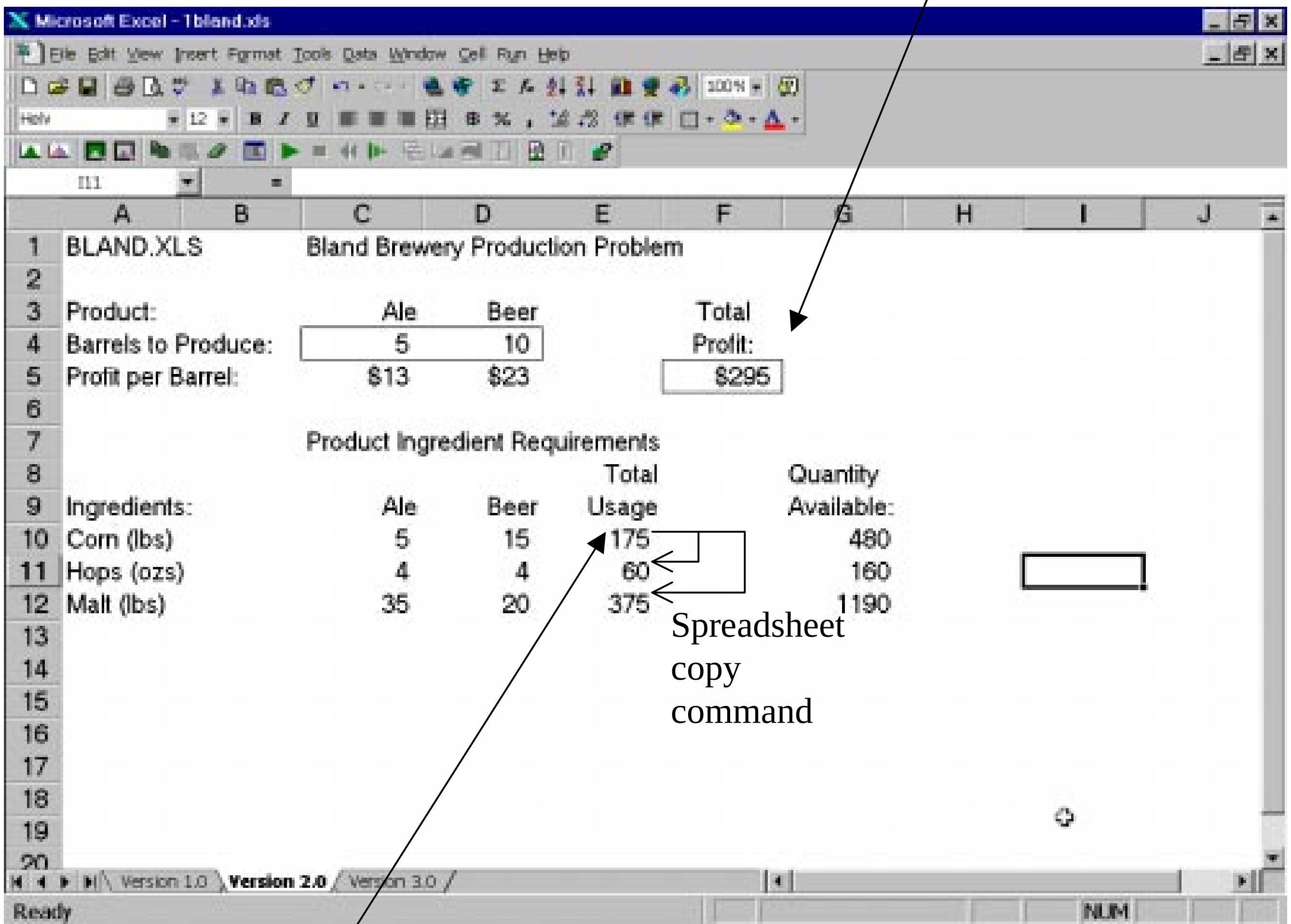


Figure 1. The preliminary spreadsheet BLAND.XLS

```
Cell F5
=SUMPRODUCT(C4:D4, C5:D5)
```




```
Cell E10
=SUMPRODUCT($C$4:$D$4, C10:D10)
```

Figure 2. The spreadsheet `BLAND.XLS` with formulas

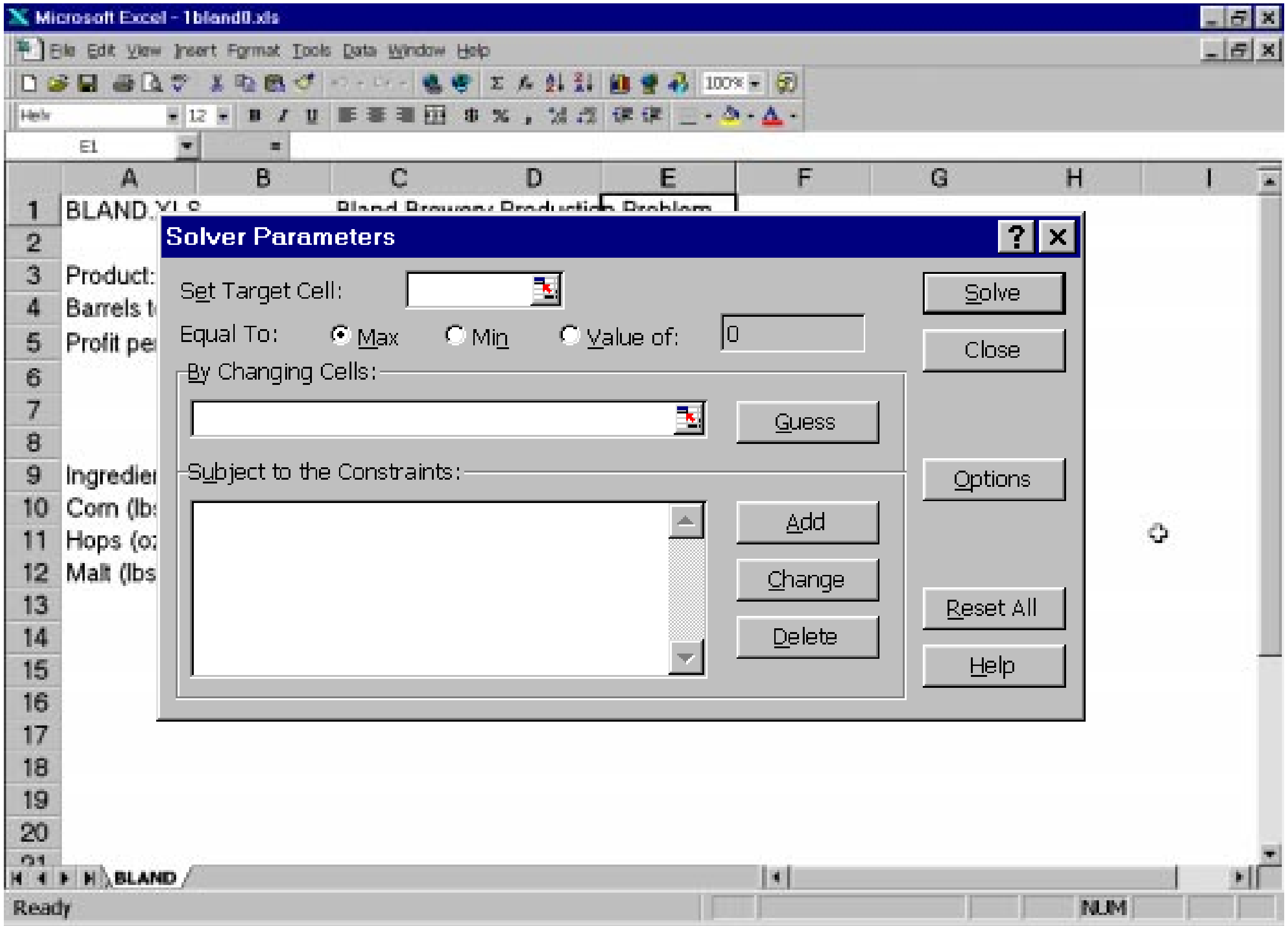


Figure 3. The *Solver Parameters* dialog box

A description of the Excel spreadsheet optimizer is given in the reading "An introduction to Spreadsheet Optimization using Excel".

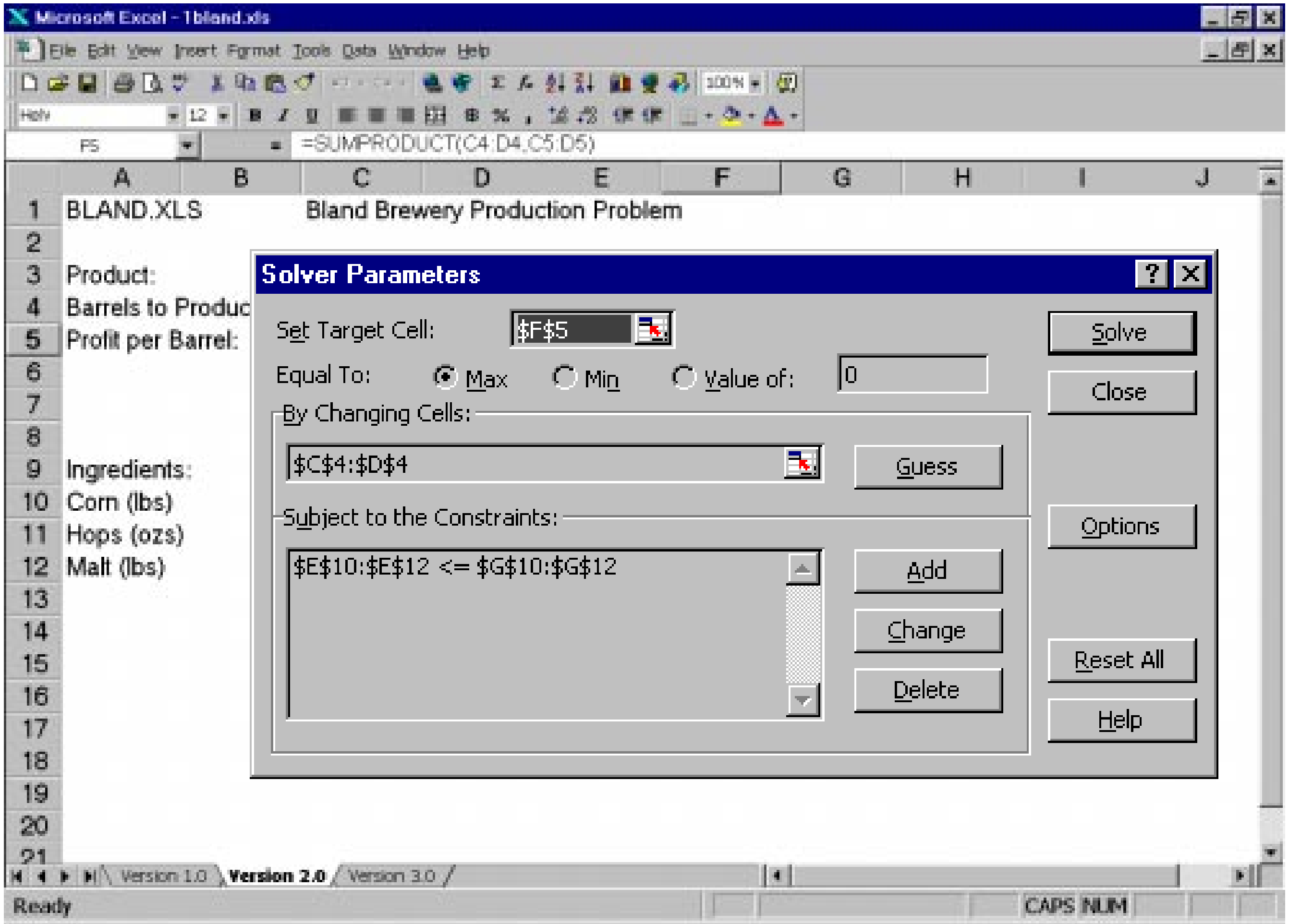


Figure 4. The *Solver Parameters* dialog box with constraints added

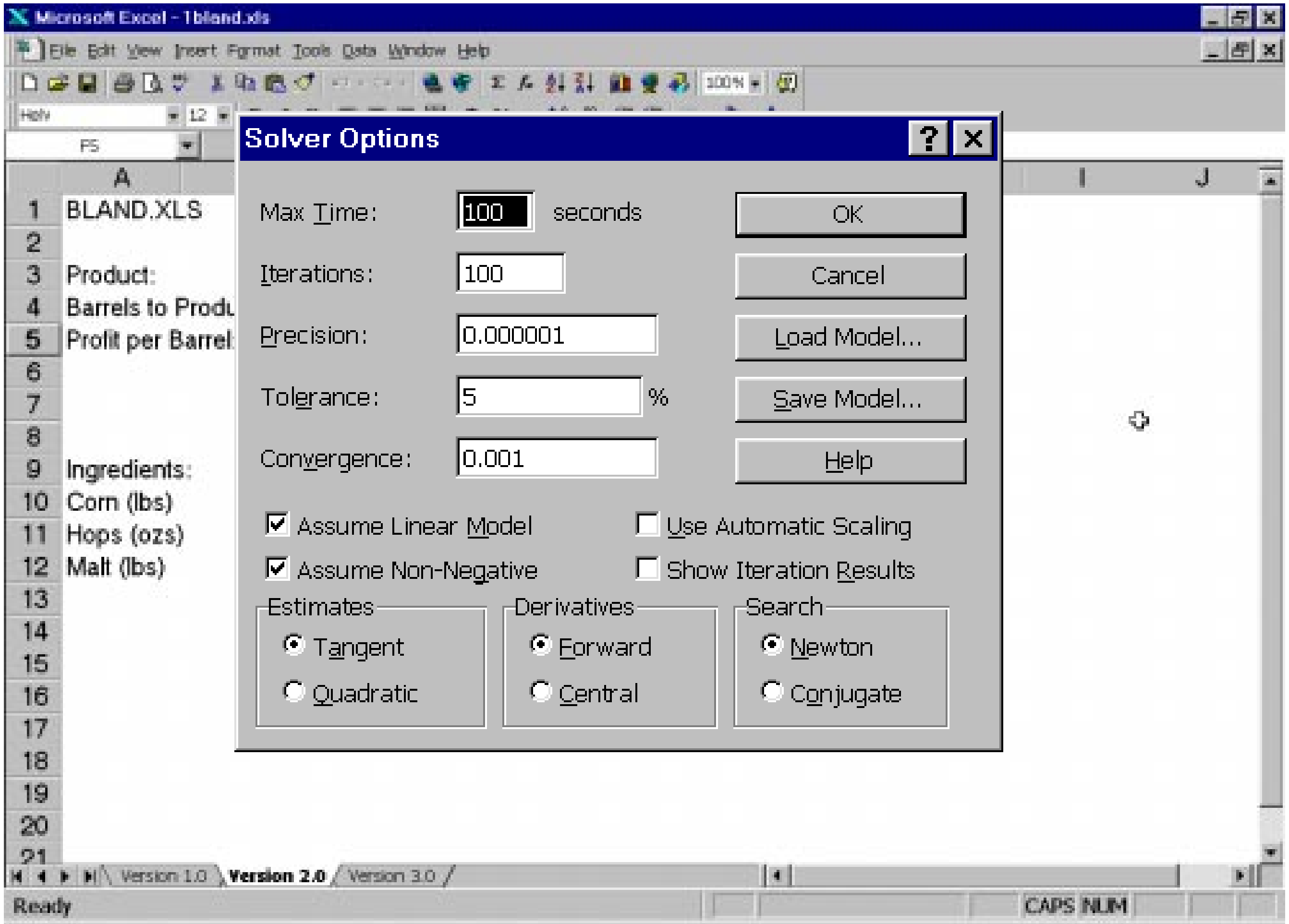


Figure 5. The *Solver Options* dialog box

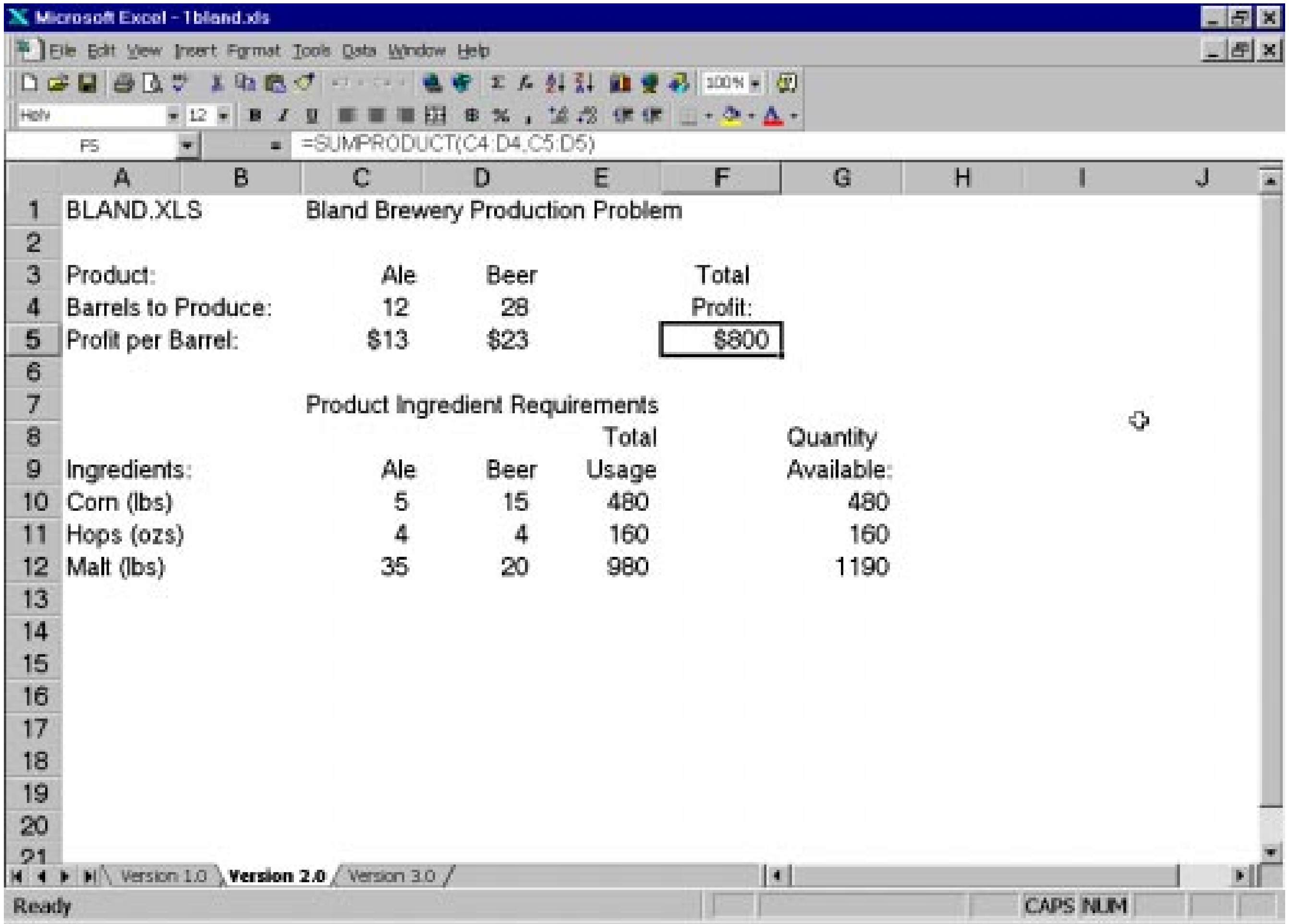
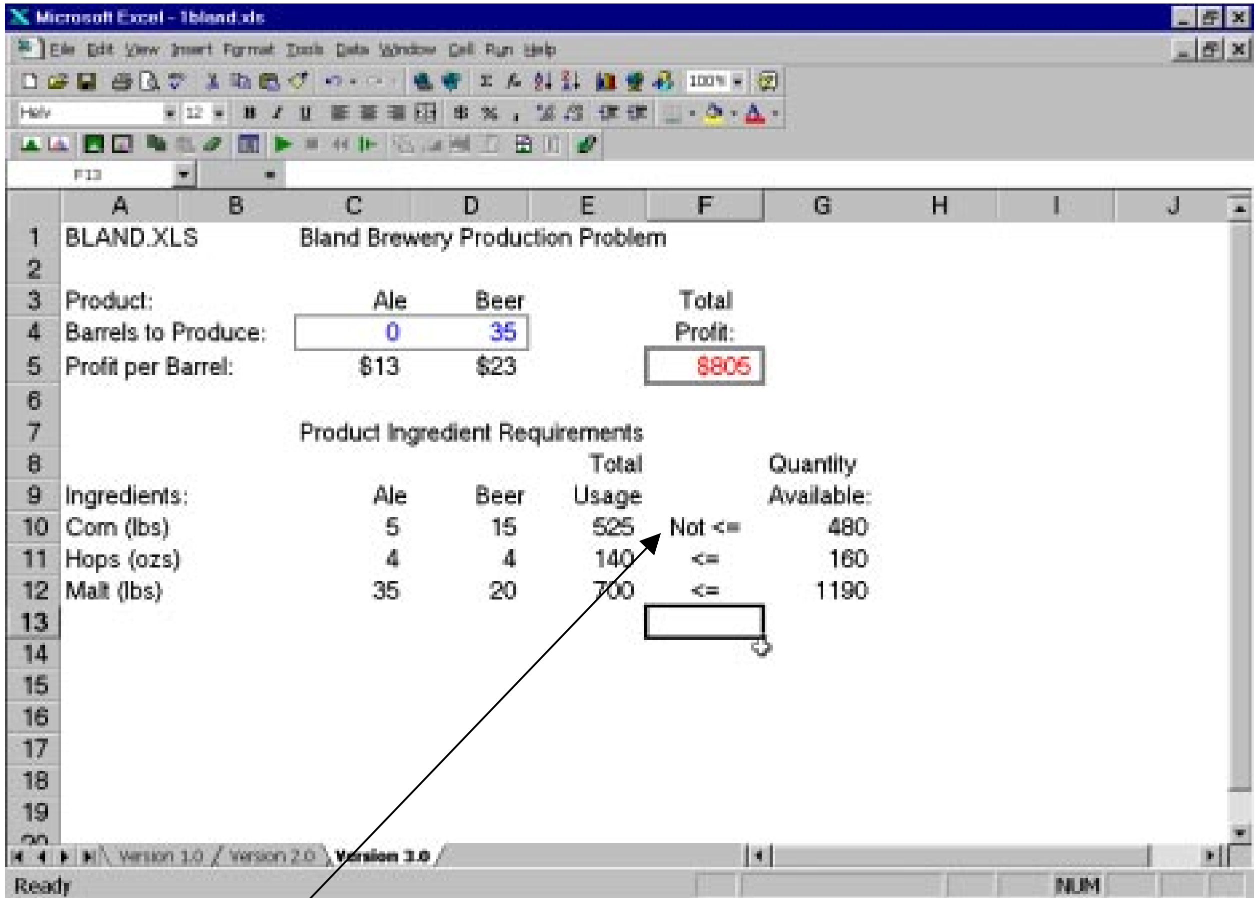


Figure 6. The spreadsheet after optimizing




```
Cell F10
=IF(E10<=G10+0.00001, “<=“, “NOT <=“)
```

Figure 7. The spreadsheet with constraints indicated

Bland Brewery LP Standard Notation

- *Decision Variables*

  Let A = # of barrels of ale to produce, and

  B = # of barrels of beer to produce.

Note: Use suggestive (mnemonic) variable names for readability.

- Bland Brewery Linear Program

$$\max\ 13A + 23B \quad \text{(Profit)}$$

subject to

$$
\begin{aligned}
&\text{(corn)} & 5A + 15B &\le 480 \\
&\text{(hops)} & 4A + 4B &\le 160 \\
&\text{(malt)} & 35A + 20B &\le 1190 \\
&\text{(nonnegativity)} & A, B &\ge 0
\end{aligned}
$$

Objective Function Coefficients

Right hand sides

Coefficients

Terminology

- *Feasible and Infeasible Solutions*
  - A production plan (*A,B*) that satisfies all of the constraints is called a *feasible solution*.
  - For example, in the Bland Brewery LP, the solution (*A*=10, *B*=10) is feasible. The production plan (*A*=40, *B*=10) is not feasible, i.e. it is *infeasible* because the hops and malt constraints are *violated*.

- *Optimal Solution*
  - For a maximization (respectively, minimization) problem, an *optimal solution* is a feasible solution that has the largest (respectively, smallest) objective function value among all feasible solutions.
  - The optimal solution for the Bland Brewery production model is (*A*=12, *B*=28). This means that Bland's optimal production plan is to produce 12 barrels of ale and 28 barrels of beer. The optimal objective function value is $800.

Assumptions in a Linear Program

- Continuity: the decision variables are continuous, i.e., fractional values are allowed.
- Proportionality: for example, it takes twice as much hops to make twice as much beer or ale; there are no economies of scale.
- Additivity: profit is the *sum* of the profit contributions from ale and beer.

  In short, the objective function and constraints must be *linear*. For example, $13A + 23B$ is a linear function of A and B. The functions $13A^2 + 23AB$ and $\log(A) + \cos(B)$ are *nonlinear functions*. The function $\max(A,0)$ is *not differentiable* at A=0 and =IF(A< 5,0,10) is a *discontinuous* function.

Allowable variations:

- Objective function can be maximized or minimized.
- Constraints can be $\geq$, $\leq$, or $=$.
- Noninteger or integer coefficients and righthand sides are allowed.
- Negative or positive coefficients and righthand sides are allowed.

Summary

- Understand LP terminology: decision variables, objective function, constraints, feasible and infeasible solutions, optimal solution
- Formulate simple linear programs
- Solve simple linear programs in a spreadsheet

Preparation for next class

- Formulate and solve the “Shelby Shelving” case (in the readings book or on pp.68-69 in the W&A text). Prepare to discuss the case in class, but do not write up a formal solution.
- Readings: “ An Introduction to Spread-sheet Optimization Using Excel” in the readings book. Read Chapter 1 and Chapter 2.1-2.4, 2.6, 2.8 and Chapter 4.1, 4.2 in the W&A text.
- Optional readings: “OR Brews Success for San Miguel” and “Logistics Steps Onto Retail Battlefield” in the readings book.