COLUMBIA BUSINESS SCHOOL

# Lecture 8

- Retailer simulation
- Summary and Preparation for next class

# Retailer Parameters

- Stores are stocked with 2,000 units of a single fashion item
  - Management hopes for strong sales but demand is hard to predict
  - No chance for restocking the item or reallocating among stores
- Initial price is $60
- 15-week selling season
  - Goal: *maximize the revenue* from the 2,000 units.
  - Production and distribution costs have already been paid; they are sunk costs
- Four allowable price levels
  - $60 (full price), $54 (10% off), $48 (20% off), $36 (40% off)
- Management policy: price cannot be raised once it has been cut
- All items in stores that are not sold at the end of 15 weeks are sold to discounters (“jobbers”) for $25 per unit (salvage value)

# Historical Data Analysis: Item 1

| Week | Qty on on hand | Price | Sales | Average Sales |
|---|---|---|---|---|
| 1 | 2000 | 60 | 57 | |
| 2 | 1943 | 60 | 98 | |
| 3 | 1845 | 60 | 55 | 58 |
| 4 | 1790 | 60 | 41 | |
| 5 | 1749 | 60 | 60 | |
| 6 | 1689 | 60 | 39 | |
| 7 | 1650 | 54 | 106 | |
| 8 | 1544 | 54 | 55 | |
| 9 | 1489 | 54 | 64 | |
| 10 | 1425 | 54 | 43 | |
| 11 | 1382 | 54 | 131 | 76 |
| 12 | 1251 | 54 | 112 | |
| 13 | 1139 | 54 | 62 | |
| 14 | 1077 | 54 | 31 | |
| 15 | 1046 | 54 | 80 | |
| 16 | 966 | | | |

- After the price is cut from $60 to $54, average weekly sales increase from 58 to 76. This represents a 30% jump in demand for item 1.

# Historical Data Analysis: Item 6

| Week | Qty on on hand | Price | Sales | Average Sales |
|---|---|---|---|---|
| 1 | 2000 | 60 | 94 | |
| 2 | 1906 | 60 | 85 | |
| 3 | 1821 | 60 | 170 | |
| 4 | 1651 | 60 | 155 | 114 |
| 5 | 1496 | 60 | 126 | |
| 6 | 1370 | 60 | 64 | |
| 7 | 1306 | 60 | 105 | |
| 8 | 1201 | 48 | 229 | |
| 9 | 972 | 48 | 253 | |
| 10 | 719 | 48 | 179 | |
| 11 | 540 | 48 | 163 | |
| 12 | 377 | 48 | 223 | 209 |
| 13 | 154 | 48 | 154 | |
| 14 | 0 | 48 | 0 | |
| 15 | 0 | 48 | 0 | |
| 16 | 0 | | | |

- The average weekly sales at $60 are 114. The average weekly sales at $48 are 209. This average is taken over weeks 8 -12 only. The average-weekly-sales increase represents an 83% jump in demand. A similar analysis can be done for each of the other items.

# Historical Data Analysis: All items

| Item | Average weekly sales at given price (ignores weeks with stockouts) 60 | 54 | 48 | 36 | Demand jump | Average jump | Incremental jump |
|---|---|---|---|---|---|---|---|
| 1 | 58 | 76 | | | 1.30 | 1.31 | 31% |
| 2 | 108 | 144 | | | 1.34 | | |
| 3 | 59 | 82 | | | 1.39 | | |
| 4 | 61 | 78 | | | 1.27 | | |
| 5 | 93 | 114 | | | 1.23 | | |
| 6 | 114 | | 209 | | 1.83 | 1.73 | 32% |
| 7 | 67 | | 120 | | 1.77 | | |
| 8 | 53 | | 97 | | 1.83 | | |
| 9 | 74 | | 132 | | 1.79 | | |
| 10 | 67 | | 97 | | 1.44 | | |
| 11 | 100 | | | 264 | 2.63 | 2.81 | 62% |
| 12 | 64 | | | 189 | 2.94 | | |
| 13 | 66 | | | 197 | 3.00 | | |
| 14 | 61 | | | 164 | 2.67 | | |
| 15 | 62 | | | 175 | 2.81 | | |

- Items 6 and 11 ran out of stock. Weeks with stockouts were removed from the average-sales results.
- Average demand at full price differs considerably across items. However, as we see here, the responsiveness of demand to price changes is similar across items.

# Retailer Pricing Strategy

- Because of the complexity of this problem, the optimal strategy is difficult to determine. However, we can try to find good strategies for the real problem by finding optimal strategies for a simpler problem.
- Thus, suppose demand is known and deterministic (i.e., not random). To illustrate, suppose weekly demand at full price is 125. Using the historical data analysis, demand at the other price levels is:

| | Price levels | | | |
|---|---|---|---|---|
| | 60 | 54 | 48 | 36 |
| Demand multiplier | 1 | 1.31 | 1.73 | 2.81 |
| Weekly demand | 125 | 164 | 216 | 351 |

- For these known levels of demand, what pricing strategy maximizes total revenue?

We can formulate a linear program to solve this problem.

- *Decision Variables*: Let

  $x_{60}$ = # of weeks the item is sold at $60

  and define $x_{54}$, $x_{48}$, and $x_{36}$ similarly.

# Retailer Linear Program

- *Objective Function:*

  The total revenue is revenue from sales plus revenue from salvage. Revenue from sales is

  $$60\ x_{60}\,(125) + 54\ x_{54}\,(164) + 48\ x_{48}\,(216) + 36\ x_{36}\,(351)\ .$$

  In order to compute the revenue from salvage, it is helpful to define an additional decision variable:

  $x_S$ = # of units sold at the salvage value of $25 .

  The revenue from salvage is simply 25 $x_S$.

- *Constraints:*
  - The constraint on total sales is “Total sales ≤ 2000,” or equivalently, “Total sales + $x_S$ = 2000.” This gives

    $$125\ x_{60} + 164\ x_{54} + 216\ x_{48} + 351\ x_{36} + x_S = 2000\ .$$

  - The selling season is at most 15 weeks:

    $$x_{60} + x_{54} + x_{48} + x_{36} \le 15\ .$$

    Note: The selling season could be less than 15 weeks if the item sells out.

  - The initial price of the item is $60, i.e., the item sells at full price for at least one week

    $$x_{60} \ge 1\ .$$

# Retailer Linear Program (continued)

- If weekly demand at full price is 125, the complete retailer linear program is:

$$\max\ 60\, x_{60}\,(125) + 54\, x_{54}\,(164) + 48\, x_{48}\,(216) + 36\, x_{36}\,(351) + 25\, x_S.$$

subject to:

- Total-sales constraint:

$$125\, x_{60} + 164\, x_{54} + 216\, x_{48} + 351\, x_{36} + x_S = 2000\ .$$

- Selling-season constraint:

$$x_{60} + x_{54} + x_{48} + x_{36} \leq 15\ .$$

- Initial-price constraint:

$$x_{60} \geq 1$$

- Nonnegativity: All variables $\geq 0$

# Retailer Optimized Spreadsheet

| A | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | RET_LP.XLS | | Retailer Linear Programming Spreadsheet | | | | | | |
| 2 | | | | | | | | | |
| 3 | Weekly demand at full price........ | | | 125 | | | | | |
| 4 | Initial quantity on hand................ | | | 2000 | | | | | |
| 5 | Salvage value........................... | | | 25 | | | | | |
| 6 | | | | | | | | | |
| 7 | Price levels................ | | 60 | 54 | 48 | 36 | | | |
| 8 | Demand multipliers..... | | 1 | 1.31 | 1.73 | 2.81 | | | |
| 9 | Avg. weekly demand.. | | 125 | 163.8 | 216.3 | 351.3 | | | |
| 10 | | | | | | | Total | Constraint | |
| 11 | No. of weeks at price | | 11.8 | 3.2 | 0.0 | 0.0 | 15 | <= | 15 |
| 12 | Total sold at price | | 1471.7742 | 528.2258 | 0 | 0 | 2000 | <= | 2000 |
| 13 | | | | | | | | | |
| 14 | Revenue computations: | | | | | | | | |
| 15 | Revenue from sales: | | 116830.65 | | | Additional Constraint: | | | |
| 16 | Revenue fom salvage: | | 0.0 | | | No. of weeks at full price >= 1? | | | |
| 17 | Total revenue | | $ 116,831 | | | | | | |
| 18 | | | | | | | | | |

+$D$3*F8

+F9*F11

=SUMPRODUCT(C7:F7,C12:F12)

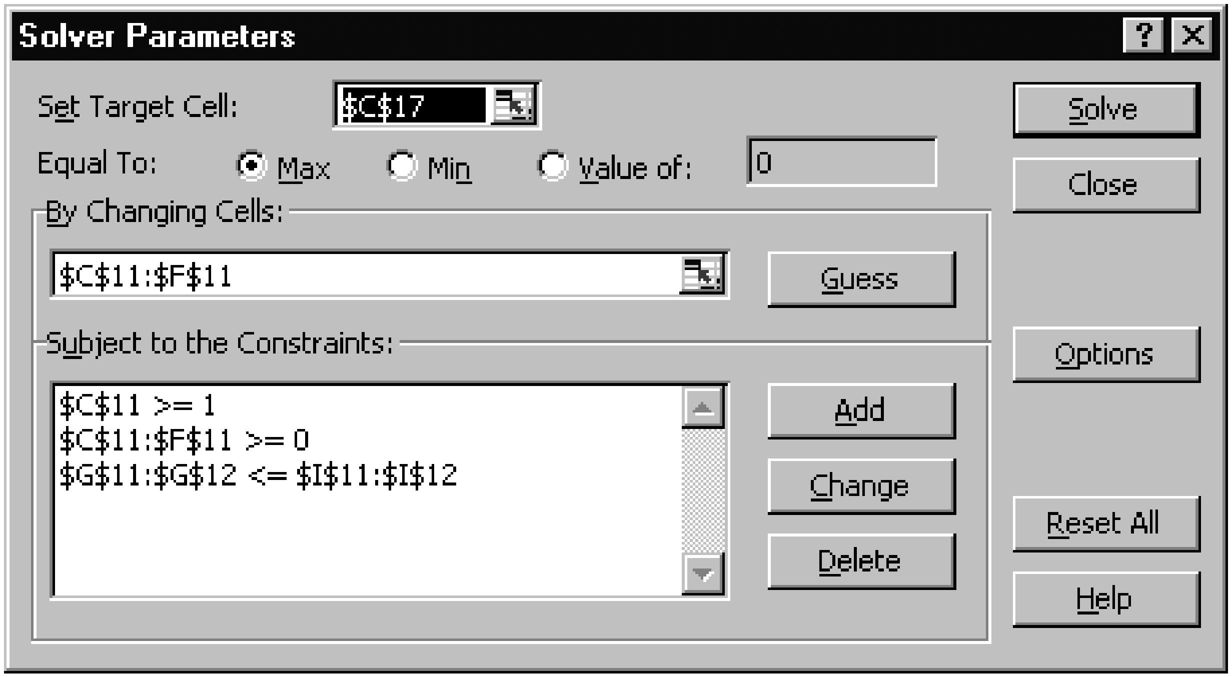
+C15+C16

(D4-G12)*D5

i.e.,(quantity salvaged)*salvage value

- The optimal solution is to sell at full price for 11.8 weeks and at 10% off for 3.2 weeks. The total revenue is $116,831.
- The decision variables could be restricted to take on integer values only. However, this is of little consequence because the exact demand rates are not known with certainty anyway.

# Retailer LP Solver Parameters




The Solver Parameters for the Retailer linear program.

# Retailer Optimization Results

| Demand at full price | $x_{60}$ | $x_{54}$ | $x_{48}$ | $x_{36}$ | $x_S$ | Total Revenue |
|---|---|---|---|---|---|---|
| 125 | 11.8 | 3.2 | 0.0 | 0.0 | 0 | 116,831 |
| 120 | 9.6 | 5.4 | 0.0 | 0.0 | 0 | 114,929 |
| 110 | 4.7 | 10.3 | 0.0 | 0.0 | 0 | 111,126 |
| 100 | 1.0 | 12.4 | 1.6 | 0.0 | 0 | 106,969 |
| 90 | 1.0 | 7.1 | 6.9 | 0.0 | 0 | 102,129 |
| 80 | 1.0 | 0.5 | 13.5 | 0.0 | 0 | 97,289 |
| 70 | 1.0 | 0.0 | 14.0 | 0.0 | 235 | 91,444 |
| 60 | 1.0 | 0.0 | 14.0 | 0.0 | 487 | 85,524 |
| 50 | 1.0 | 0.0 | 14.0 | 0.0 | 739 | 79,603 |
| 40 | 1.0 | 0.0 | 14.0 | 0.0 | 991 | 73,682 |

- If demand is sufficiently large (e.g., at least 80 per week at full price), the optimal solution is to cut the price to sell the last unit (number 2000) at the end of week 15.
- It is never optimal to cut the price to \$36. Why?

# Which price cuts are beneficial?

- Suppose that weekly demand at $60 is 100. If the price is kept at $60, 1500 units will be sold over the 15-week selling season. The total revenue is

$$60(1500) + 25(2000 - 1500) = 90{,}000 + 12{,}500$$
$$= 102{,}500 .$$

- Suppose that weekly demand at 10% off ($54) is 115 - a 15% increase. Is the retailer better off with a constant $60 price or a constant $54 price?

  If the price is set at $54, 1725 (= 15(115)) units will be sold over the 15-week selling season. The total revenue is

$$54(1725) + 25(2000 - 1725) = 93{,}150 + 6{,}875$$
$$= 100{,}025 .$$

- The retailer would be worse off! With a price cut of 10%, a demand increase of 15% is *not* enough to produce an increase in revenue. Why? Because of the salvage value.

# Beneficial price cuts (continued)

- The complete stock of 2000 units can be sold for at least the salvage value of \$50,000 (= 2,000 $\times$ 25). The goal of maximizing revenue can be restated to maximize the *incremental* revenue over this amount.

- The incremental revenue per unit at the initial price is \$35 (= 60 - 25). At a price of 10% off, the incremental revenue per unit is \$29 (= 54 - 25). The incremental revenue decreases by a factor of

  $$0.83 = 29/35 .$$

  Demand must increase by a factor of

  $$1.207 = 35/29$$

  to make up for the price cut.

- Indeed, if demand increased to 120.7 at a price of \$54, 1811 (=15(120.7)) units would be sold over the 15 week selling season. The total revenue would be

  $$54(1811) + 25(2000 - 1811) = 97{,}767 + 4{,}738$$
  $$= 102{,}500 ,$$

  the *same* as selling for 15 weeks at \$60.

# Breakeven Demand Increases

- To summarize, if the price is cut by 10% (from \$60 to \$54), demand must increase by 21% (from 100 to 121) to break even in revenue. The analysis can be repeated for the other price cuts:

| Price | Actual demand increase | Breakeven increase |
|---|---|---|
| 10% off (\$54) | 31% | 21% |
| 20% off (\$48) | 73% | 52% |
| 40% off (\$36) | 181% | 218% |

- For a price cut from \$60 to \$36, incremental revenue decreases by a factor of

$$0.31 = 11/35\ .$$

  Demand must increase by a factor of

$$3.18 = 35/11$$

  to make up for the price cut. But demand only increases by a factor of 2.81.

  *Moral*: Under these circumstances, it is not optimal to cut the price to \$36.

# Effect of Uncertainty in Demand

- The retailer linear program gave the optimal solution with known and deterministic demand. If demand at full price were 90, the optimal linear programming solution is $x_{60}$ = 1.0, $x_{54}$ = 7.1 and $x_{48}$ = 6.9. Suppose we didn't know the actual demand, and cut the price (from $54 to $48) one week later. How would the total revenue change? How would the revenue change if we cut the price one week earlier?

| Demand at full price | $x_{60}$ | $x_{54}$ | $x_{48}$ | $x_{36}$ | $x_S$ | Total Revenue |
|---|---|---|---|---|---|---|
| 90 | 1.0 | 7.1 | 6.9 | 0.0 | 0 | 102,129 |
| 90 | 1.0 | 8.1 | 5.9 | 0.0 | 38 | 101,967 |
| 90 | 1.0 | 6.1 | 7.6 | 0.0 | 0 | 101,422 |

  - The optimal strategy gives a revenue of $102,129.
  - Cutting 1 week too late gives a revenue of $101,967, or $162 less than optimal.
  - Cutting 1 week too early gives a revenue of $101,422, or $707 less than optimal.
  - If the true demand were not known, there is a greater risk of cutting the price too early compared to too late.

# Effect of Uncertainty in Demand (continued)

- The same analysis can be repeated for other demand levels. Suppose demand at full price is 120. The optimal linear programming solution is to keep the price at \$60 for 9.6 weeks and at \$54 for 5.4 weeks. Cutting one week later or one week earlier gives:

| Demand at full price | $x_{60}$ | $x_{54}$ | $x_{48}$ | $x_{36}$ | $x_S$ | Total Revenue |
|---|---|---|---|---|---|---|
| 120 | 9.6 | 5.4 | 0.0 | 0.0 | 0 | 114,929 |
| 120 | 10.6 | 4.4 | 0.0 | 0.0 | 37 | 114,570 |
| 120 | 8.6 | 6.1 | 0.0 | 0.0 | 0 | 114,209 |

  - The optimal strategy gives a revenue of \$114,929.
  - Cutting 1 week too late gives a revenue of \$114,570, or \$359 less than optimal.
  - Cutting 1 week too early gives a revenue of \$114,209, or \$720 less than optimal.
  - If the true demand were not known, there is a greater risk of cutting the price too early compared to too late.

# Retailer Pricing Strategy

The preceding analysis suggests some pricing strategy guidelines to follow when demand is random and unknown:

- Only use “beneficial” price cuts. In this case, don’t cut the price to $36.
- If demand is sufficiently great, time price cuts in order to run out of stock at the end of the 15 week selling season.
- With uncertain demand, cutting the price too late is less risky than cutting the price too early.
- Once a price cut is taken, it cannot be rescinded. With uncertain demand, this also argues for cutting later rather than earlier.

How would a different salvage value affect these conclusions?

# Related Simulation Applications

- The retailer simulation illustrates how simulation can be used as a *training* tool. Other examples include:
- Foreign-exchange market simulator
  - Used to train traders and market makers in the basic elements of foreign-exchange markets
- Soapmaker
  - Used to train production managers in the basic tradeoffs in production scheduling and inventory control
- MARKSTRAT 2
  - Simulation of firm-wide marketing strategy
  - Decisions include product design, distribution, pricing, advertising, and sales-force allocation
- The Stanford Bank Game
  - Simulates the management of a large commercial bank

These simulation programs are used in upper-level finance, operations, and marketing courses.

# Summary

- Retailer illustrates an application of simulation to yield management
- Simulation as a training tool
- Optimization can be used to develop reasonable pricing strategies

# For next class

- Read Chapter 12, pp.581-594 in the W&A text.
- At this point, you should make sure to install Crystal Ball on your computer.