



# COLUMBIA BUSINESS SCHOOL

## B6015: Decision Models

Spring 2000

### Solutions to Sample Questions from Past Final Exams

#### 1. Briton Paper Company

The solution is in the Excel file: BRITON.XLS available on the web page.

- a)
- |                     |                              |
|---------------------|------------------------------|
| Formula in cell C26 | =MAX(C\$12*C34,\$D6)         |
| Formula in cell G6  | =SUMPRODUCT(C16:G19,C26:G29) |
| Formula in cell H17 | =SUM(C17:G17)                |
| Formula in cell D20 | =SUM(D16:D19)                |

b) Objective function:

G6 =SUMPRODUCT(C16:G19,C26:G29) = Total shipping cost

Total shipping cost equals the number of trucks assigned to each destination (by carrier) multiplied by the cost per delivery (which is carrier-specific) summed up over all destinations and carriers pairs.

Decision variables:

C16:G19. Each decision variable represents the number of trucks driven by each carrier to each destination.

Constraints:

C20:G20 = C22:G22. The number of truckloads required at each destination should equal the number shipped.

H16:H19 ≤ J16:J19. The number of truckloads sent by each carrier must be less than or equal to the number of trucks available.

C16:G19. The decision variables must be non-negative.

- c) The correct answer is Carrier 3. In Figure 2 (the sensitivity report), look at the shadow price of the constraints in cells H16 through H19. Specifically, look at which one has the most negative shadow price. This is carrier 3 with a shadow price of -250. The allowable increase is 2, so this means that if we increase the number of trucks available from carrier 3 from 6 to 7 trucks, we will see a decrease in the objective function of \$250.
- d) See Figure 2 (the Constraints part). The new optimal cost is:  $\$34,957 + 2(\$843) = \$36,643$ . The shadow price of cell C20 is 843 and the allowable increase is 3. Thus an increase in 2 units in the number of truckloads required will lead to an increase in cost of  $2(\$843)$ .
- e) This new rate will not affect the optimized spreadsheet. Carrier 2 to destination a has an Allowable Decrease (in the sensitivity report under Adjustable Cells, cell C17) of 213. This means that the cost of carrier 2 shipping to a can be reduced by \$213 without affecting the optimal solution. The current cost of carrier 2 shipping to a is \$1056. A decrease of \$213 corresponds to a new cost of  $\$1056 - \$213 = \$843$ . In cost per mile this becomes:  $\$843/600$

= \$1.405. Thus a cost per mile of \$1.46 is within the allowable decrease, and therefore for such a cost the optimal solution stays the same.

g) A constraint of this type can be written as follows:

$$=SUMPRODUCT(C17:G17,C27:G27) \geq 0.25*G6$$

The total amount paid to carrier 2 is at least 25% of the total shipping cost.

h) Here we ignore all modifications made in previous questions. For this problem, you need to solve the model in Excel.

New optimal plan:

	A	b	c	d	e	Total
Carrier 1	0	0	0	1	4	5
Carrier 2	0	3	0	0	1	4
Carrier 3	0	0	2	4	0	6
Carrier 4	4	1	0	0	0	5
Total	4	4	2	5	5	20

Cost table used (cells C26:G29 in the spreadsheet):

	a	b	C	d	e
Carrier 1	888	2175	1764	3660	400
Carrier 2	1056	2190	2148	4890	500
Carrier 3	624	2055	1512	3510	500
Carrier 4	828	2175	1908	4320	500

New optimal cost: \$34,881

## 2. NiftyCalc. Inc

To estimate the expected return on the junk bond, we will use simulation.

a) We simulate the cash flow for six trials. To simulate the outcome of the company we will use the following rule (any rule you use is correct as long as it ends up with the right probability for each outcome):

Let X be a uniform random number.

If  $X \leq 0.55$  then the outcome is a "success".

If  $0.55 < X \leq 0.8$  then the outcome is a "marginal failure"

If  $0.8 < X$  then the outcome is a "total failure".

Trial	X	Outcome	Prime Rate	Bond Value	Bond Return
1	0.10	success	10.95%	$100 + (100)(5)(10.95\%) = \$154.75$	54.75%
2	0.74	marg. failure	13.05%	\$70	-30.00%

3	0.53	success	9.00%	$100 + (100)(5)(9.00\%) = \$145.00$	45.00%
4	0.95	total failure	14.79%	\$30	-70.00%
5	0.33	success	13.80%	$100 + (100)(5)(13.80\%) = \$169.00$	69.00%
6	0.19	success	18.00%	$100 + (100)(5)(18.00\%) = \$190.00$	90.00%

[Note it is also correct if you do not use one of the normal numbers from Table 2 in trials 2 and 4 where one is not needed. Your answers on these questions will be different, but with a large number of trials the difference will be negligible.]

[Also, there are many ways one could define the event of a success, marginal failure, and total failure. For example, if  $X \leq 0.2$ , then total failure. if  $0.2 < X \leq 0.75$ , then success. if  $0.75 < X$ , then marginal failure. If you do it this way, then you will also get different answers, but it will be correct.]

- b) Based on the simulation (6 trials), we estimate the expected return of the bond to be:  
 Expected Return =  $(1/6)(54.75\% - 30\% + 45\% - 70\% + 69\% + 90\%) = 26.46\%$ .

- c) The standard deviation of the bond return is: 62.42% (I used =STDEV()). Thus a 90% confidence interval for the bond's expected return can be written as follows:

$$26.46 \pm 1.645 (62.42/\sqrt{6}) = 26.46\% \pm 41.92\%.$$

[Note this answer is an approximation since the number of trials is small (less than 30) and the returns are not, in this case, normally distributed.]

To get an estimated expected return within  $\pm 1\%$  (at the same 90% confidence level), you should use the formula. If  $\sigma$  is the population standard deviation, and you want a 90%

confidence interval of width  $2W$ , you need to have  $n$  at least  $\left(\frac{1.645 \sigma}{W}\right)^2$ .

In this case, we need:  $n \geq \left(\frac{1.645 \sigma}{1\%}\right)^2$ . We will use  $\sigma = 62.42\%$  as an estimate of the population standard deviation. The answer is:  $n \geq 10,543.4$  or  $n \geq 10,544$ .

- d) To estimate the probability that the cash flow is at least \$105, we look at our six trials and determine the proportion of time cash flow was above \$105. It happened 4 out of 6 times for an estimate of  $4/6 = 66.7\%$ .
- e) After running Crystal Ball (see the file NIFTY.XLS) we get an average or expected return of 15.22%. We can estimate the probability that cash flow exceeds \$105 in the Crystal Ball forecast window: it is 58.40%. (Enter a 5% return on the left hand side of the frequency chart.) Another way to get the probability (and also a confidence interval for the estimate) is to make a forecast cell with the formula =IF(F8>105,1,0) and then give the mean of the forecast cell.