COLUMBIA BUSINESS SCHOOL

Lecture 2

- TransportCo Distribution Problem
- Advertising Problem
- Shelby Shelving Case
- Summary and Preparation for next class

Shelby Shelving Decision Model

- *Decision Variables*:

  Let *S* = # of Model *S* shelves to produce, and

  LX = # of Model *LX* shelves to produce.

- To specify the objective function, we need to be able to compute net profit for any production plan (*S, LX*). Case information:

	S	*LX*
Selling Price	1800	2100
Standard cost	1839	2045
Profit contribution	-39	55

$$\Rightarrow \text{Net Profit} = -39\,S + 55\,LX \qquad (1)$$

  So for the current production plan of *S* = 400 and *LX* = 1400, we get

  Net profit = $61,400.

- Is equation (1) correct?

- Equation (1) is not correct (although it does give the correct net profit for the *current* production plan). Why? Because the standard costs are based on the current production plan and they do not correctly account for the fixed costs for different production plans.

- For example, what is the net profit for the production plan $S = LX = 0$? Since

  Net Profit = Revenue - Variable cost - Fixed cost

  and Fixed cost = 385,000, the Net profit is -$385,000. But equation (1) incorrectly gives

  Net profit = -39 S + 55 LX = 0

  To derive a correct formula for net profit, we must separate the fixed and variable costs.

Profit Contribution Calculation

	Model *S*	Model *LX*
a) Selling price	1800	2100
b) Direct materials	1000	1200
c) Direct labor	175	210
d) Variable overhead	365	445
e) Profit contribution (e = a-b-c-d)	260	245

- The correct objective function is

  Net profit = 260 S + 245 LX - 385,000 (2)

Shelby Shelving LP

- *Decision Variables:*

  Let S = # of Model S shelves to produce, and

  LX = # of Model LX shelves to produce.

- Shelby Shelving Linear Program

$$\max \ 260\,S + 245\,LX - 385{,}000 \qquad \text{(Net Profit)}$$

subject to:

(S assembly)	S	≤ 1900
(LX assembly)	LX	≤ 1400
(Stamping)	$0.3\,S + 0.3\,LX$	≤ 800
(Forming)	$0.25\,S + 0.5\,LX$	≤ 800
(Nonnegativity)	S, LX	≥ 0

Note: Net profit = Profit - Fixed cost, but since fixed costs are a constant in the objective function, maximizing Profit or Net Profit will give the same optimal solution (although the objective function values will be different).

Spreadsheet Solution

Decision Variables

Objective Function
`+H3-H4`

A	A	B	C	D	E	F	G	H	I
1	SHELBY.XLS		**Shelby Shelving Company**						
2									
3			Model S	Model LX		Gross profit		653,250.00	
4	Production per month		1900	650		Fixed cost		385,000.00	
5	Variable profit contribution		$260	$245		Net profit		$268,250.00	
6									
7	Selling price		1800	2100					
8	Direct materials		1000	1200					
9	Direct labor		175	210					
10	Variable overhead		365	445					
11	Variable profit contribution		260	245					
12									
13					Total		Total		
14			Usage per unit		Used	Constraint	Available		
15	Model S assembly		1	0	1900	<=	1900		
16	Model LX assembly		0	1	650	<=	1400		
17	Stamping (hours)		0.3	0.3	765	<=	800		
18	Forming (hours)		0.25	0.5	800	<=	800		
19									

`SUMPRODUCT($C$4:$D$4, C15:D15)`

`SUMPRODUCT(C4:D4,C5:D5)`

TransportCo Distribution Problem

- TransportCo supplies goods to four customers, each requiring the following amounts:

	Demand Requirement (in units)
Nashville	25
Cleveland	35
Omaha	40
St. Louis	20

- The company has three warehouses with the following supplies available:

	Supply Available (in units)
Dallas	50
Atlanta	20
Pittsburgh	50

TransportCo Distribution Problem (cont.)

- The costs of shipping one unit from each warehouse to each customer are given by the following table:

		To			
		Nashville	Cleveland	Omaha	St. Louis
From	Dallas	$30	$55	$35	$35
From	Atlanta	$10	$35	$50	$25
From	Pittsburgh	$35	$15	$40	$30

- Construct a decision model to determine the minimum cost of supplying the customers.

TransportCo Distribution Problem Overview

- What needs to be decided?

  A distribution plan, i.e., the number of units shipped from each warehouse to each customer.

- What is the objective?

  Minimize the total shipping cost. This total shipping cost must be calculated from the decision variables.

- What are the constraints?

  Each customer must get the number of units they requested (and paid for). There are supply constraints at each warehouse.

- TransportCo optimization model in general terms:

  min Total Shipping Cost

  subject to

  - Demand requirement constraints
  - Warehouse supply constraints
  - Non-negative shipping quantities

TransportCo Distribution Model

- *Index:* Let D=Dallas, A=Atlanta, P=Pittsburgh, N=Nashville, C=Cleveland, O=Omaha and S=St. Louis.

- *Decision Variables*: Let

  X_{DN} = # of units sent from D=Dallas to N=Nashville,

  X_{DC} = # of units sent from D=Dallas to C=Cleveland,

  …..

  X_{PS} = # of units sent from P=Pittsburgh to S=St. Louis.

- *Objective Function*:

  With the decision variables we defined, the total shipping cost is:

$$30\ X_{DN} + 55\ X_{DC} + 35\ X_{DO} + 35\ X_{DS} + 10\ X_{AN} + 35\ X_{AC}$$
$$+ 50\ X_{AO} + 25\ X_{AS} + 35\ X_{PN} + 15\ X_{PC} + 40\ X_{PO} + 30\ X_{PS}$$

Demand and Supply Constraints

- *Demand Constraints*: In order to meet demand requirements at each customer, we need the following constraints:
  - For Nashville: $X_{DN} + X_{AN} + X_{PN} = 25$
  - For Cleveland: $X_{DC} + X_{AC} + X_{PC} = 35$
  - For Omaha: $X_{DO} + X_{AO} + X_{PO} = 40$
  - For St. Louis: $X_{DS} + X_{AS} + X_{PS} = 20$

- *Supply Constraints:* In order to make sure not to exceed the supply at the warehouses, we need the following constraints:
  - For Dallas: $X_{DN} + X_{DC} + X_{DO} + X_{DS} \leq 50$
  - For Atlanta: $X_{AN} + X_{AC} + X_{AO} + X_{AS} \leq 20$
  - For Pittsburgh: $X_{PN} + X_{PC} + X_{PO} + X_{PS} \leq 50$

TransportCo Linear Programming Model

$$\min \; 30X_{DN} + 55X_{DC} + 35X_{DO} + 35X_{DS} + 10X_{AN} + 35X_{AC} + 50X_{AO} + 25X_{AS} + 35X_{PN} + 15X_{PC} + 40X_{PO} + 30X_{PS}$$

subject to:

(Demand Constraints)

(Nashville) $X_{DN} + X_{AN} + X_{PN} = 25$

(Cleveland) $X_{DC} + X_{AC} + X_{PC} = 35$

(Omaha) $X_{DO} + X_{AO} + X_{PO} = 40$

(St. Louis) $X_{DS} + X_{AS} + X_{PS} = 20$

(Supply Constraints)

(Dallas) $X_{DN} + X_{DC} + X_{DO} + X_{DS} \leq 50$

(Atlanta) $X_{AN} + X_{AC} + X_{AO} + X_{AS} \leq 20$

(Pittsburgh) $X_{PN} + X_{PC} + X_{PO} + X_{PS} \leq 50$

Non-negativity: All variables ≥ 0

TransportCo Optimized Spreadsheet

Objective Function=`SUMPRODUCT(B7:E9,B13:E15)`

	A	B	C	D	E	F	G	H
1	TRANS.XLS		**TransportCo Distribution Problem**					
2								
3	Total Shipping Cost=		**$ 2,900**					
4								
5	Shipping Costs (per unit)							
6		Nashville	Cleveland	Omaha	St. Louis			
7	Dallas	$30	$55	$35	$35			
8	Atlanta	$10	$35	$50	$25			
9	Pittsburgh	$35	$15	$40	$30			
10								
11	Shipping Quantities (in units)					Total		
12		Nashville	Cleveland	Omaha	St. Louis	Shipped From		Supplies
13	Dallas	**5**	**0**	**40**	**5**	50	<=	50
14	Atlanta	**20**	**0**	**0**	**0**	20	<=	20
15	Pittsburgh	**0**	**35**	**0**	**15**	50	<=	50
16	Total Shipped to	25	35	40	20			
17		=	=	=	=			
18	Requirements	25	35	40	20			

Decision Variables

`=SUM(B13:B15)`

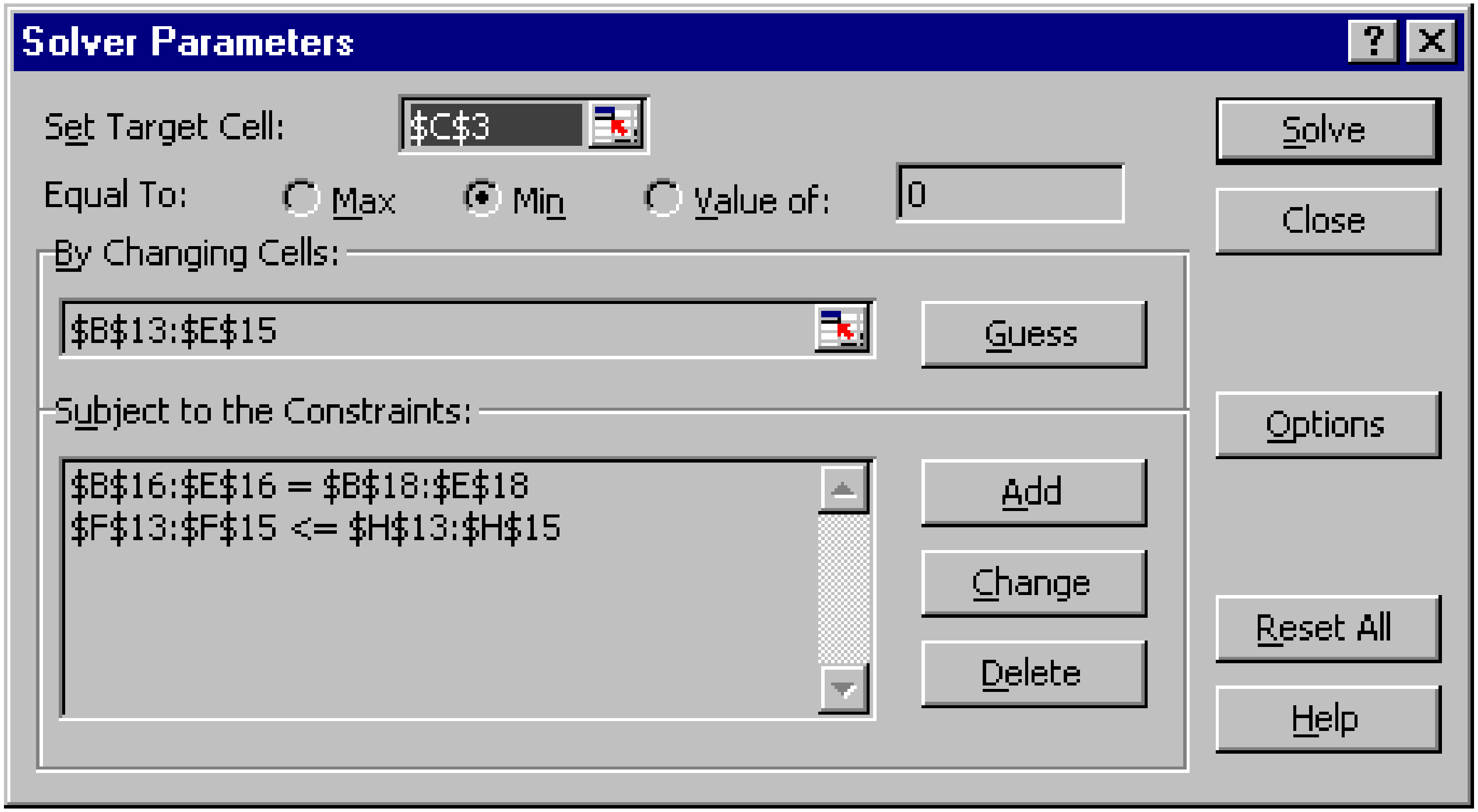
`=SUM(B13:E13)`

`=IF(ABS(B16-B18)<0.00001,”=“,”Not =“)`

`=IF(F15<=H15+0.00001,”<=“,”Not <=“)`

- The optimal solution has a total cost of $2,900.

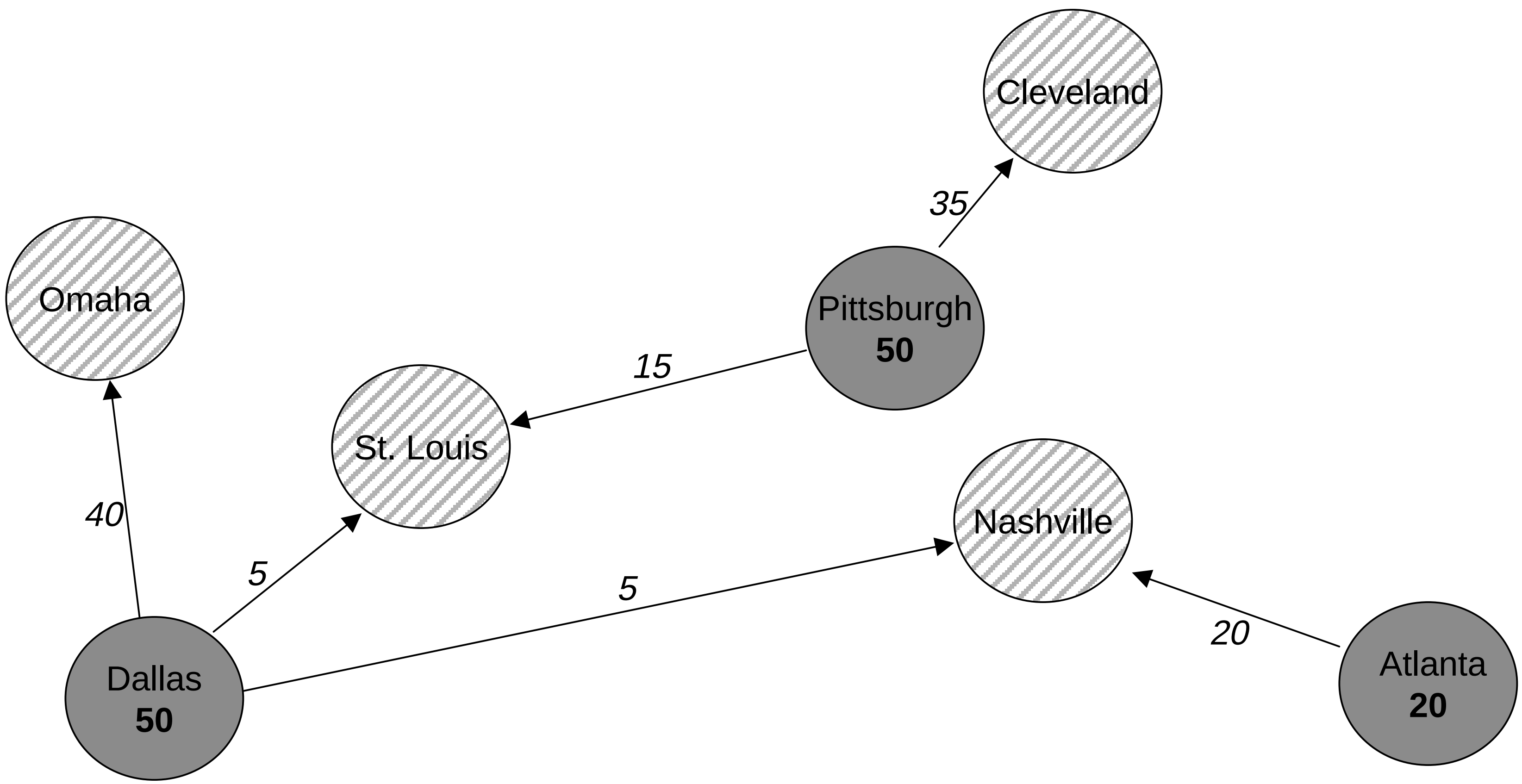
TransportCo Solver Parameters



The *Solver Parameters* dialog box with constraints added.

TransportCo Solution Summary

- The optimal solution has total cost $2,900.
- The optimal distribution plan is as follows:



Advertising Problem

- A manufacturer of personal computers would like to devise an advertising plan for the introduction of its new laptop.
- The company is considering advertising on TV (in a number of different regions) and in two magazines: Newsweek and PC Magazine.
- Market research suggests that for people whose annual income exceeds $35,000, 1.4% of those who see at least one ad will then buy the product, while for the lower income group (those making less than $35,000) only 0.6% buy the product after seeing at least one ad.

- The total advertising budget is $250,000.
- To diversify the strategy, it is stipulated that at most half of all ads can be TV ads, and at most 5 ads can be placed in any one medium (TV, Newsweek or PC Magazine).

Advertising Problem (continued)

- One unit of TV advertising costs $35,000 and has the following number of exposures per ad:
  - 1,000,000 people in the higher-income bracket (>$35,000), and
  - 1,000,000 people in the lower-income bracket (<$35,000).
- One unit of advertising in “Newsweek” costs $25,000 and has the following number of exposures per ad:
  - 750,000 people in the higher-income bracket (>$35,000), and
  - 250,000 people in the lower-income bracket (<$35,000).
- One unit of advertising in “PC Magazine” costs $15,000 and has the following number of exposures per ad:
  - 400,000 people in the higher-income bracket (>$35,000), and
  - 200,000 people in the lower-income bracket (<$35,000).

- For simplicity, we will assume that there is *no overlap* in the exposures.

- Formulate the problem of maximizing expected sales as a linear program.

Advertising Problem Overview

- What needs to be decided?

  The number of ads in each medium to place.
- What is the objective?

  Maximize the expected sales.
- What are the constraints?

  There is the budget constraint, the constraint on the number of TV ads, the limit on the number of ads in each medium, and the non-negativity constraints.
- The Advertising Problem optimization model in general terms:

  max Expected Sales

  subject to

  - Budget Constraint
  - At most half of all ads can be TV ads
  - At most 5 ads in each medium
  - Non-negative number of each ad type

Advertising Problem: Model

- *Decision Variables*: Let

  T = # of TV ads to place,

  N = # of ads to place in “Newsweek”,

  P = # of ads to place in “PC Magazine”,

- *Additional Variables*: Let

  H = millions of people in the higher-income bracket (those making at least $35,000) who see at least one ad,

  L = millions of people in the lower-income bracket (those making less than $35,000) who see at least one ad.

- *Objective Function*: With the above decision variables, the expected total sales can be determined as follows:
  - Expected number of higher-income people who buy a laptop:

    0.014 H (in millions)
  - Expected number of lower-income people who buy a laptop:

    0.006 L (in millions)
  - *Expected Total Sales (in millions of laptops)*:

    0.014 H + 0.006 L

Model Constraints

- *Budget Constraint:*

$$35{,}000\ T + 25{,}000\ N + 15{,}000\ P \leq 250{,}000$$

or equivalently, in $1,000s:

$$35\ T + 25\ N + 15\ P \leq 250.$$

- *Linking Constraints:*
  - The number of *higher*-income people seeing an ad is:

$$1{,}000{,}000\ T + 750{,}000\ N + 400{,}000\ P.$$

Since H is in millions of people this leads to the constraint:

$$H = T + 0.75\ N + 0.4\ P.$$

  - The number of *lower*-income people seeing an ad is:

$$1{,}000{,}000\ T + 250{,}000\ N + 200{,}000\ P$$

Since L is in millions of people this leads to the constraint:

$$L = T + 0.25\ N + 0.2\ P$$

Model Constraints (cont.)

- *At most half can be TV ads:*

  $$T / (T + N + P) \leq 0.5$$

  This is a *non-linear* constraint (it involves a ratio of variables). We can easily convert this to a linear constraint:

  $$T \leq 0.5 (T + N + P)$$

- *At most 5 ads in each medium:*

  $$T \leq 5$$

  $$N \leq 5$$

  $$P \leq 5$$

Advertising Problem Linear Programming Model

max $0.014\,H + 0.006\,L$

subject to:

(Budget)	$35\,T + 25\,N + 15\,P \leq 250$
(High Income)	$H = T + 0.75\,N + 0.4\,P$
(Low Income)	$L = T + 0.25\,N + 0.2\,P$
(Limit on TV)	$T \leq 0.5\,(T + N + P)$
(Other Limits)	$T \leq 5$
	$N \leq 5$
	$P \leq 5$
(Non-negativity)	$T \geq 0,\ N \geq 0,\ P \geq 0$

Advertising Problem Optimized Spreadsheet

Decision Variables

=IF(B4<=F4+0.00001,”<=“,”Not <=“)

=IF(D4<=$H$4+0.00001,”<=“,”Not <=“)

	A	B	C	D	E	F	G	H
1	MEDIA.XLS		**Advertising Problem**					
2								Max # of Ads
3		TV ads	Newsweek	PC Mag.		(0.5)(T+P+C)		per medium
4	Number of Ads	**5**	**0**	**5**		5		5
5	Other Limits	<=	<=	<=				
6	Limit on TV Ads	<=						
7								
8	Cost per Ad	$ 35,000	$ 25,000	$ 15,000				
9	Exposures per Ad (in M)					Total Cost		
10	Higher Income (>$35,000)	1.000	0.750	0.400		$250,000	<=	$ 250,000
11	Lower Income (<$35,000)	1.000	0.250	0.200				
12					Total	Prob. of		Exp. Sales
13	Total Exposures (in M)				Expos.(in M)	Purchase		(in millions)
14	Higher Income (>$35,000)	5.000	0.000	2.000	7.000	1.4%		**0.134000**
15	Lower Income (<$35,000)	5.000	0.000	1.000	6.000	0.6%		

=0.5*SUM(B4:D4)

=SUMPRODUCT(B8:D8,B4:D4)

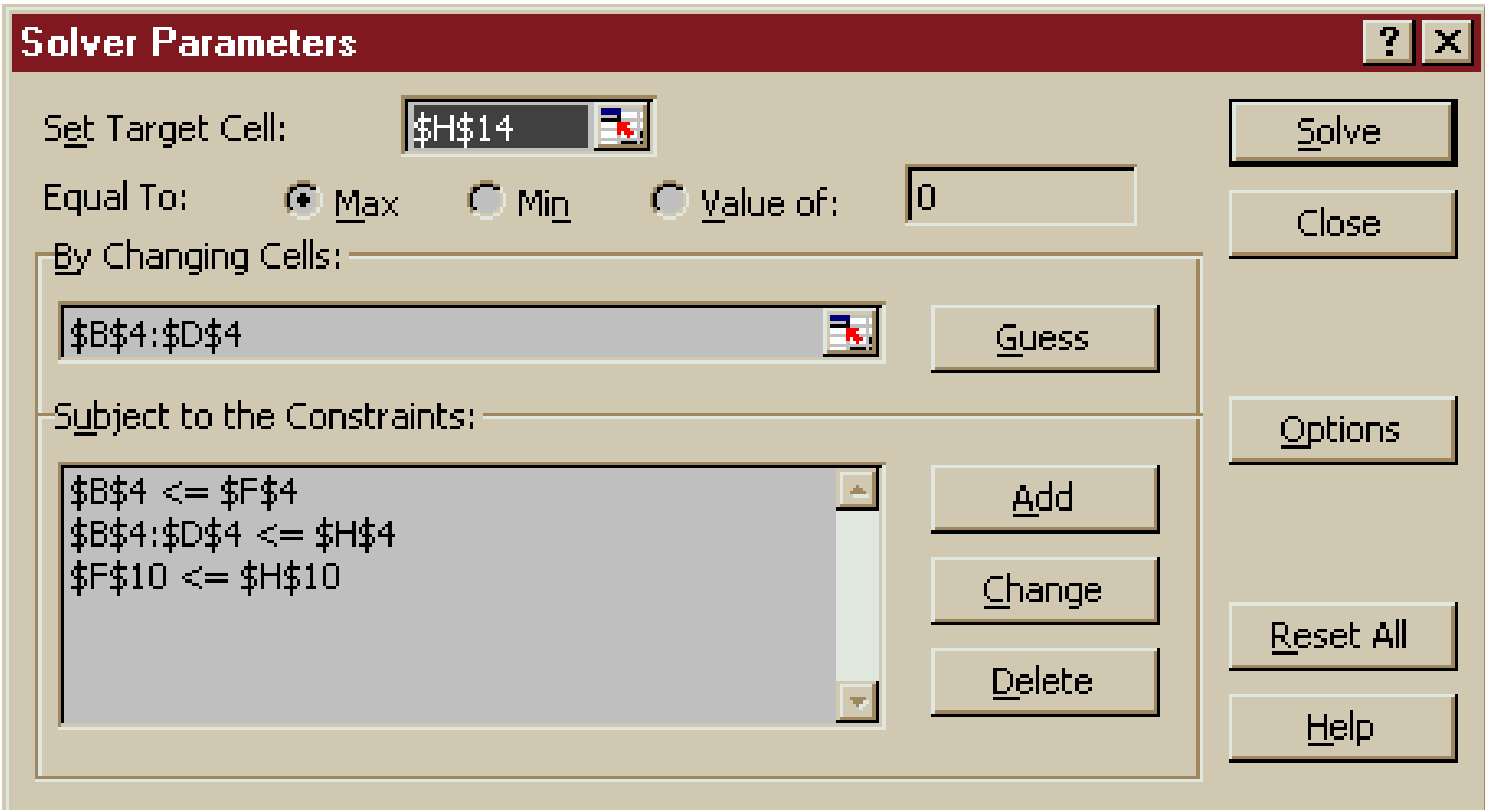
=SUM(B15:D15)

=B10*B$4 and copied to range B14:D15

=SUMPRODUCT(E14:E15,F14:F15)

- The optimal solution has 134,000 in expected sales (in laptops).

Advertising Problem: Solver Parameters



Solver Parameters for the Advertising Problem

Advertising Problem: Solution Summary

- The optimal solution suggests 5 TV ads and 5 "PC Magazine" ads, with about 134,000 in expected sales (in laptops).
- This strategy will get 7 million higher-income exposures and 6 million lower-income exposures.
- Therefore we expect (7 million)(1.4%)= 98,000 laptops will be sold to the higher-income people, and (6 million)(0.6%)= 36,000 laptops will be sold to lower-income people.
- The total cost is $250,000, i.e., the entire budget is used up.

How can the model be improved?

- In reality there might be some overlap in the exposures (for example, some people will see both the TV ad and the PC Magazine ads).
- The probability of purchase is most likely not as simple as described here.

Summary

- Examples of Two Formulations
- Lesson from Shelby Shelving: Be careful about fixed versus variable costs

For next class

- Read and think about the case Petromor: The Morombian State Oil Company. (Prepare to discuss the case in class, but do not write up a formal solution.)
- Read Chapter 4.4 in the W&A text.
- Load the SolverTable add-in to Excel. The needed files are available at the course web-page, where there are also instructions on how to do this.
- *Optional reading*: "Graphical Analysis" in the readings book.