Decision Models

COLUMBIA BUSINESS SCHOOL

Lecture 9

- Simulation
  - → Introduction
  - → Simple example
  - → Generating uniform and normal random numbers on a spreadsheet
  - → Introduction to Crystal Ball

- Risk management simulation example
  - → Insuring against oil price increases
  - → Analysis of results

- Summary and Preparation for next class

Simulation

What is Monte Carlo simulation?
- A model that mimics the behavior of a (stochastic) system
- Mathematically described the system using a set of equations
- Some of the variables in the equations may be random variables.

Familiar Examples
- Flight simulator
- MARKSTRAT
- Video poker

Typical applications
- Telecommunications and computer network design
- Security pricing (e.g., oil insurance contract)
- Timing traffic lights on city streets
- Factory analysis and design

Class examples: applications of simulation to *yield management* (Retailer and Ski Jacket cases), *risk management*, and *security pricing.*

Simulation Pros and Cons

Advantages

- Fast: can simulate years of real time in seconds of computer time

- Robust: can obtain results for very complex systems

- "Off-Line" Experimental tool: Can modify a simulation and observe the results without changing the real system (e.g., effect of an expensive new machine, changes in traffic light timing)

Disadvantages

- Costly: programming effort can be large (in $ and time)

- Brute force: must evaluate each possible action, i.e., simulation is not an optimization technique

- Inexact: simulation output is given in the form of a distribution of results
  - → Statistical analysis is necessary to interpret simulation output

Coin Flipping Example

Flip a coin which has a probability of 0.35 for "Heads" and 0.65 for "Tails." If the outcome is "Heads" you win $3. If the outcome is "Tails" you pay $1. What is the expected profit (or loss) of this game?

Spreadsheet set-up:

	A	B	C
1	COIN.XLS		
2			
3	Random #	Outcome	Profit
4	0.735	Tail	−1

Cell A4: Define cell A4 to be a random sample from a uniform (0,1) distribution. This can be done using =RAND() in Excel. We'll show how to do it with Crystal Ball.

Cell B4: If Random # < 0.35 then "Head" else "Tail." In spreadsheet cell B4 use the formula

$$=IF(A4 < 0.35, \text{"Head"}, \text{"Tail"}).$$

Cell C4: If Outcome = "Head" then Profit = 3 else Profit = −1. In spreadsheet cell C4 use the formula

$$=IF(A4 < 0.35, 3, -1).$$

Crystal Ball Simulation

Steps to running a simulation using Crystal Ball:

(0) Load Crystal Ball (if it is not loaded automatically, from the main Excel menu choose "Tools" and then "Add-Ins")

(1) Define assumption cell(s) (in this case, cell A4 should be specified as a uniform (0,1) distribution)

(2) Define forecast cell(s) (in this case, cell C4 is the only forecast cell)

(3) Set simulation run preferences (in this case, we will set a random number seed and specify the number of trials to run)

(4) Run the simulation

Crystal Ball toolbar:

1. Define assumption   3. Run preferences

2. Define forecast   4. Start simulation
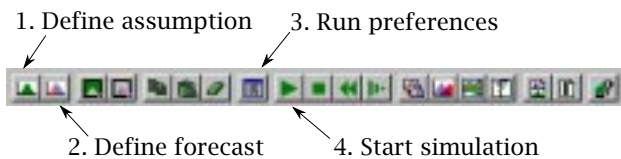
We'll show the details of these steps next.

Simulation Step 1. Define Assumption Cell

First make sure there is a number in cell A4 (it cannot be blank or contain a formula). Then move the cursor to cell A4 and click on the Crystal Ball "Define Assumption" icon.
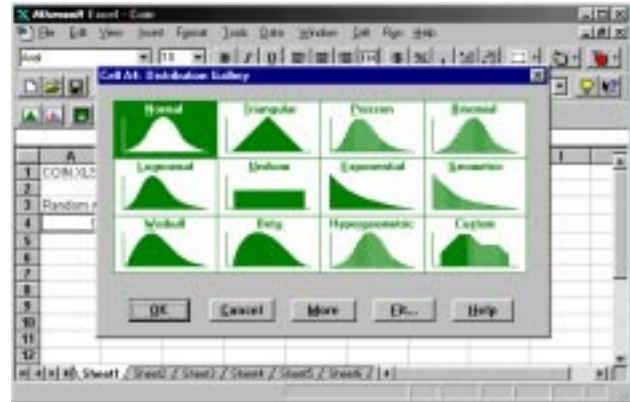
Choose the uniform distribution from the distribution gallery.

Simulation Step 1. (continued)

After clicking on uniform, the uniform distribution window will appear:
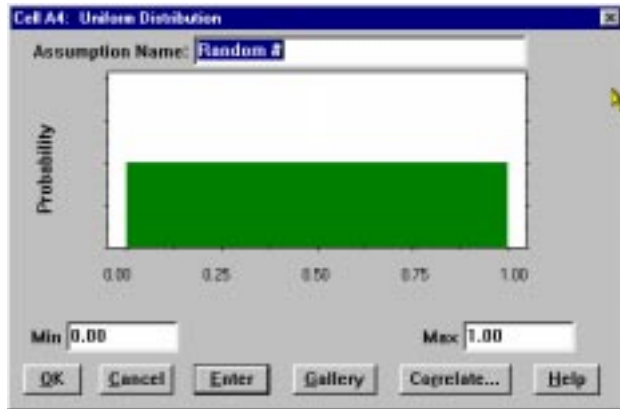
Set the "Min" of the uniform to 0 and the "Max" to 1. Then press "OK."

Simulation Step 2. Define Forecast Cell

Move the cursor to cell C4. Then click on the Crystal Ball "Define Forecast" icon. When the "Define Forecast" window appears, change the Forecast Name from "Tail" to "Profit." The window should now look as shown below:
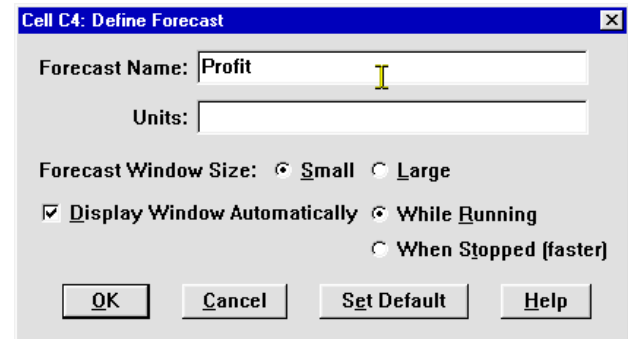
Press "OK" to return to the spreadsheet.
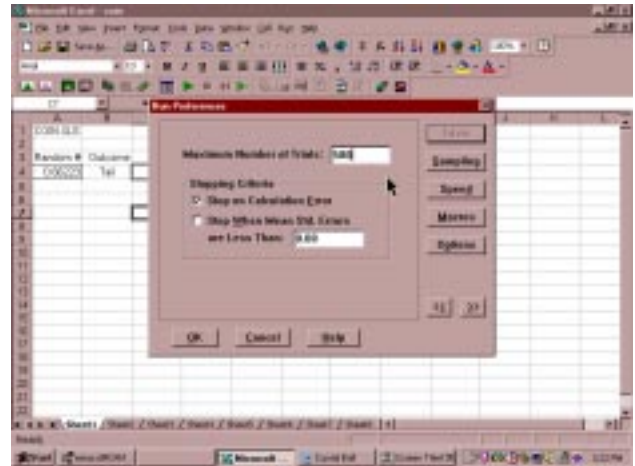
Simulation Step 3. Set Run Preferences

Click on the Crystal Ball "Run Preferences" icon. Check the box "Use Same Sequence of Random Numbers" and enter an "Initial Seed Value" of 123.
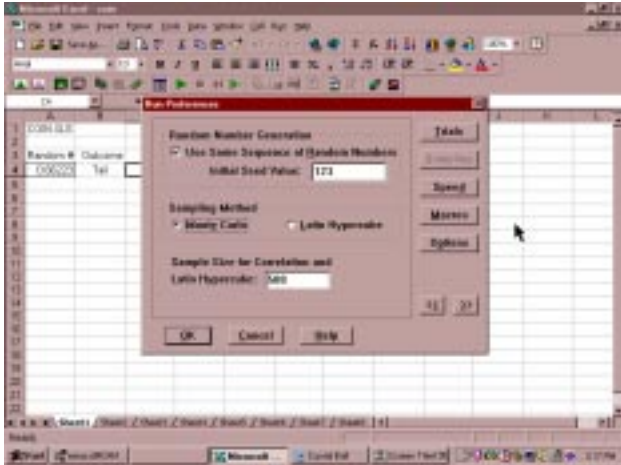
Now press the "Trials" in the upper right of the Run Preferences dialog box.

Simulation Step 3. Set Run Preferences (cont)

Set the "Maximum Number of Trials" to 500.

Now press "OK" to return to the spreadsheet.

Simulation Step 3. Comments

Setting an initial seed value allows us to duplicate the results of a simulation, i.e., to use the same sequence of random numbers. For example, if everyone in the class uses the same seed and the same number of trials, the simulation results should be *identical*. Thus, setting a seed is useful for debugging a simulation spreadsheet.

Caveats: To get identical results, it is important not only to set the same seed, but to make sure that no other Crystal Ball spreadsheets are open at the same time (all open spreadsheets are simulated when "Run" is pressed). Also, if there are multiple assumption cells, these need to be specified in the same order to get identical results.

Of course there is nothing special about setting the seed to "123." Any value will do, but it is useful for everyone to use the same seed to check our results.
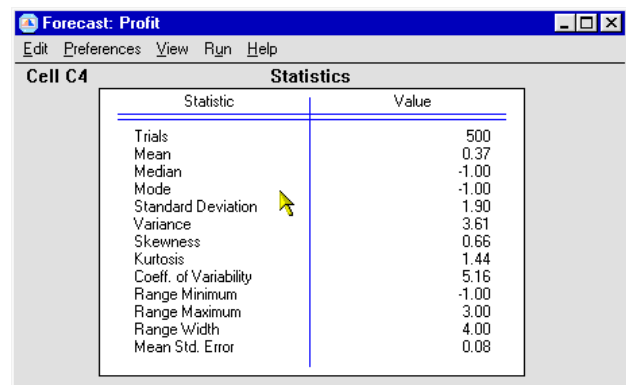
In this spreadsheet, the "Maximum Number of Trials" sets the number of coin flips to simulate. Entering a larger number of trials will increase the time required to run the simulation, but it will also give more accurate results.

Simulation Step 4. Run the Simulation

Click on the Crystal Ball "Start Simulation" icon. The profit forecast window will appear, and the number of trials simulated will show in the bottom left corner of the Excel window.

When the simulation is complete, Crystal Ball will display the message "Maximum number of trials reached." Click on "OK."

In the forecast window, click on "View" and then "Statistics" to display the numerical results shown below:

Forecast: Profit		
Edit Preferences View Run Help		
Cell C4	**Statistics**	
Statistic		Value
Trials		500
Mean		0.37
Median		-1.00
Mode		-1.00
Standard Deviation		1.90
Variance		3.61
Skewness		0.66
Kurtosis		1.44
Coeff. of Variability		5.16
Range Minimum		-1.00
Range Maximum		3.00
Range Width		4.00
Mean Std. Error		0.08

Simulation Results

The 500 trial Crystal Ball simulation using the seed of 123 gives an estimate of the mean profit of $0.37.

The number $0.37 is only an *estimate* of the true mean profit from the coin flipping game. The standard error of the mean is 0.08. This indicates that a 95% confidence interval for the true mean profit is approximately $0.37 \pm 2(0.08)$.

To get a better estimate using simulation, we could increase the number of simulation trials, and continue the simulation run.

For this example, though, the true mean profit can be computed exactly. The calculation is:

Expected profit $= 0.35(3) + 0.65(-1) = \$0.40.$

For many realistic applications, there are no formulas to compute the true expected profit. In these cases, simulation is often the only viable method to estimate the expected profit.
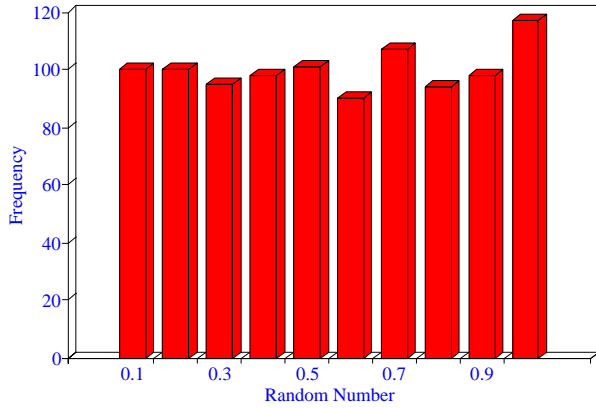
Histogram of Simulated Uniform Distribution
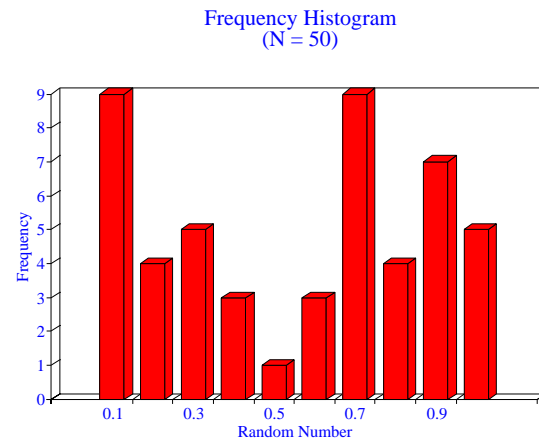
For a small sample (e.g., 50 random numbers) the frequency histogram might not look uniform at all.

Histogram of Simulated Uniform Distribution

Frequency Histogram
(N = 1000)

As the sample size is increased (e.g., to 1000 random numbers in this case) the frequency histogram will look more nearly uniform.

Histogram of Simulated Normal Distribution

Frequency Histogram
(N = 50)

For a small sample (e.g., 50 random numbers) the frequency histogram might not look normally distributed at all.

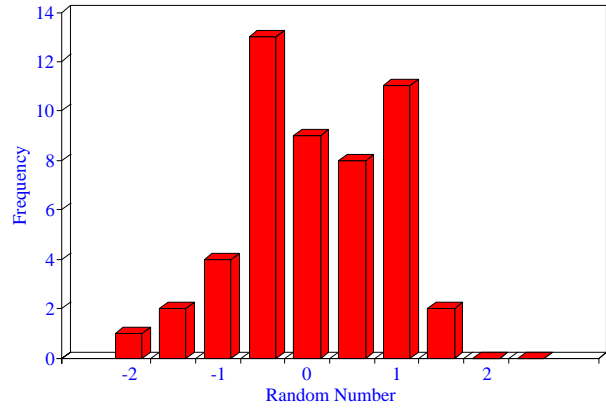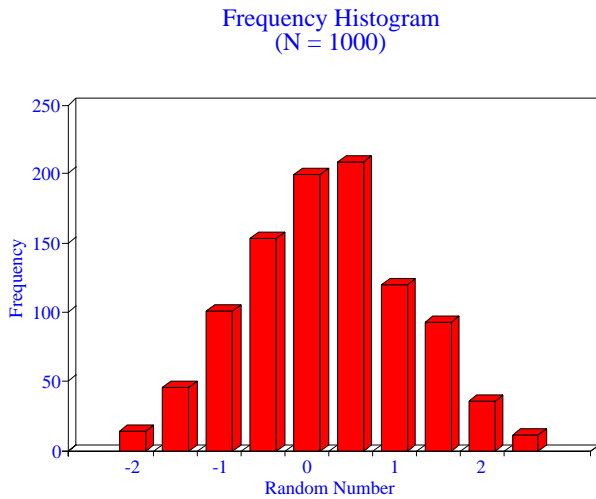## Histogram of Simulated Normal Distribution

Frequency Histogram
(N = 1000)

As the sample size is increased (e.g., to 1000 random numbers in this case) the frequency histogram will look more nearly normal.

What sample size is large enough? Compute a *confidence interval* for the desired result. Use enough samples so that you are comfortable with the resulting confidence interval.

Daily Crude Oil Prices

Crude Oil Price
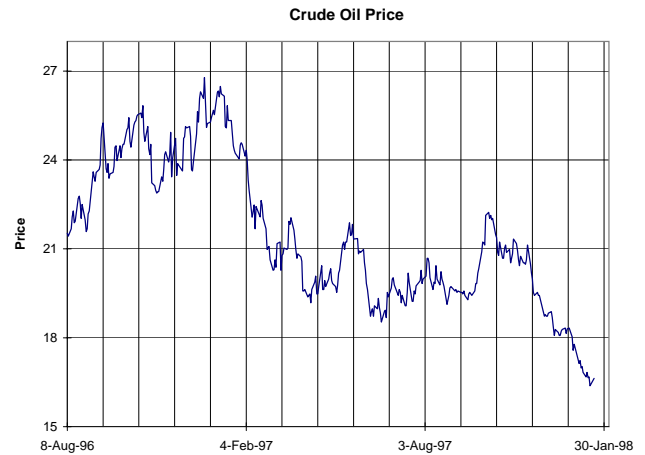
○ Lakefield notices that the price of oil is at its lowest point since at least August 1996.

○ Lakefield is worried that the recent price trend could change, and so it would like to lock in the low price for the coming month.

○ However, if the price continues to drop over the coming month, Lakefield would like to benefit from the price decline.

Crude Oil Price Risk Management

○ Trusty BankCo insures petroleum refining
  companies against upswings in the price of crude
  oil

  → Lakefield Oil purchases 50,000 barrels of crude
    oil daily for its refining operations
  → Lakefield Oil wants to "lock in" a price for
    buying crude in the coming month, while …
  → … maintaining the possibility of saving money
    should the price of crude drop

○ The insurance contract works as follows:
  → First day of the month: TrustyCo and Lakefield
    agree on a *target price* for crude
  → Last day of the month: the average daily price
    of crude for the month is calculated
  → If the average daily price exceeds the target
    price, TrustyCo pays Lakefield

    Average Daily Price − Target Price

  for each barrel of crude purchased.
  → If the average daily price is less than the target
    price, no money changes hands.

What is a Fair Price for Oil Price Protection?

○ Suppose on the first day of the month:

  → The spot price of crude oil is $17 per barrel
  → The *target price* that Lakefield wants is also $17
    per barrel
  → The month has 20 trading (refining) days

○ In order to determine a fair price for the oil
  insurance contract:
  1. Simulate the evolution of the crude oil price
     over the next 20 days
  2. Calculate the average oil price for the month
  3. Calculate the payment (possibly zero) from
     TrustCo to Lakefield
  Repeat steps 1–3 and average the results, i.e.,
  compute the average payment from TrustCo to
  Lakefield.

○ For step 1, we need a model of potential oil price
  movements over the *coming* month
  → In order to develop such a model, let's examine
    the histogram of daily crude oil returns

Histogram of Daily Crude Oil Returns

Histogram of Crude Oil Return
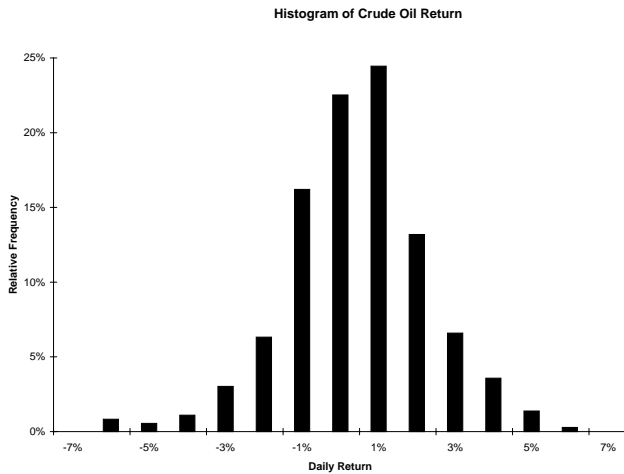
- The histogram summarizes daily oil returns over the period August 1996 through January 1998.

- The daily returns appear to be approximately normally distributed.

- The mean daily return (-0.05%) is not significantly different from zero.

- The standard deviation of the most recent 40 daily returns is 1.15%.

Simulation of the Insurance Contract

- Time index: $t = 1, 2, \ldots, 20$

- Assume that the daily returns $R_1, R_2, \ldots, R_{20}$ are independent and identically normally distributed, each with mean $\mu = 0\%$ and $\sigma = 1.15\%$.

- Daily oil prices are denoted $P_0, P_1, P_2, \ldots, P_{20}$, where

$$P_0 = 17$$
$$P_1 = P_0(1 + R_1)$$
$$P_2 = P_1(1 + R_2)$$
$$\vdots$$
$$P_{20} = P_{19}(1 + R_{20})$$

- To compute the payment from TrustCo to Lakefield, calculate
  - → Average price $A = (P_1 + \cdots + P_{20})/20$
  - → Payment $= \max(0, A - 17)$

Spreadsheet Implementation

=AVERAGE(C9:C28)

=C3*(1+B9)

|	A	B	C	D	E	F
1	ASIANOIL.XLS					
2						
3	Initial Oil Price		$17.00			
4	Target Price		$17.00			
5	Mean daily oil return		0.00%			
6	Std dev daily oil return		1.15%			
7						
8	Day	Oil Return	Oil Price		Average Price	
9	1	-0.77%	16.87		17.49	
10	2	1.28%	17.08			
11	3	1.70%	17.37		Payout	
12	4	0.97%	17.54		0.49	
13	5	-0.54%	17.45			
14	6	-0.97%	17.28			
15	7	0.61%	17.39			
16	8	0.43%	17.46			
17	9	-0.86%	17.31			
18	10	-0.55%	17.22			
19	11	1.86%	17.54			
20	12	-0.73%	17.41			
21	13	0.82%	17.55			
22	14	0.83%	17.70			
23	15	-0.08%	17.68			
24	16	-0.05%	17.68			
25	17	-0.09%	17.66			
26	18	0.77%	17.79			
27	19	0.97%	17.97			
28	20	-0.45%	17.89			

=MAX(E9-C4,0)

Crystal Ball Spreadsheet Simulation

(1) Define the assumption cells B9:B28. These should be normally distributed with mean C5 and standard deviation C6.

(2) Define the forecast cell E12.

(3) Set simulation run preferences. For this example, set the random number seed to 123, set the maximum number of trials to 500 (and use the Monte Carlo sampling method).

A quick way to define multiple assumption cells:

(1a) Define assumption cell B9 (move cursor to cell B9 and then click on the Crystal Ball "Define Assumption" icon)

(1b) With the cursor on cell B9, click on the Crystal Ball "Copy Data" icon

(1c) Highlight cells B10:B28. Then click on the Crystal Ball "Paste Data" icon

1b. Copy data

1a. Define assumption   1c. Paste data
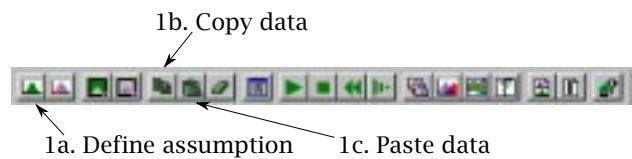
Computing a Confidence Interval for the Fair Value

After the simulation is run, the forecast cell E12 (Payout) has:
- Mean 0.2159
- Std deviation 0.3368
- Std error 0.0151

Hence, a fair value for the insurance contract is approximately 21.6 ¢ per barrel.

What is a 95% confidence interval for the fair value based on the simulation ouput?

General formula for a $(1 - \alpha)$ confidence interval:

$$\overline{X} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}.$$

For a 95% confidence interval: $\alpha = 0.05$, and $z_{\alpha/2} = 1.96$. In general, $z_{\alpha/2}$ can be computed in a spreadsheet using `NORMSINV(1 - α/2)`, e.g., `NORMINV(0.975) = 1.959961`.

For our example, $\overline{X} = 0.2159$, $n = 500$, $s = 0.3368$, and the 95% confidence interval is:

$$0.2159 \pm 1.96 \frac{0.3368}{\sqrt{500}},$$

i.e., the 95% confidence interval is

$$[0.186, 0.245].$$

The Oil Insurance Contract

- The insurance contract in this example in called an *Asian option*.

- In practice, simulation is a typical approach to estimating its value.

- The actual calculations used by derivatives professionals vary slightly from what we have done:

  → the average payout should be discounted by the interest rate over the twenty days, i.e., the fair value is the average discounted payout (not the average payout).

  → we should not use zero for the mean rate of return of the oil (although this will have no discernable impact in this example).

- For a complete treatment of option pricing, see the book *Options, Futures, and Other Derivatives*, by John C. Hull, third edition, Prentice-Hall, 1997.

Summary

∘ Create a spreadsheet simulation using
  Crystal Ball
  → Uniform random numbers
  → Normal random numbers

∘ Analyze simulation output
  → Averages
  → Standard Errors
  → Histograms

For next class

∘ Optional reading: Chapter 12.5–12.12 and
  Chapter 13 in the W&A text.