COLUMBIA
BUSINESS
SCHOOL

*Decision Models*

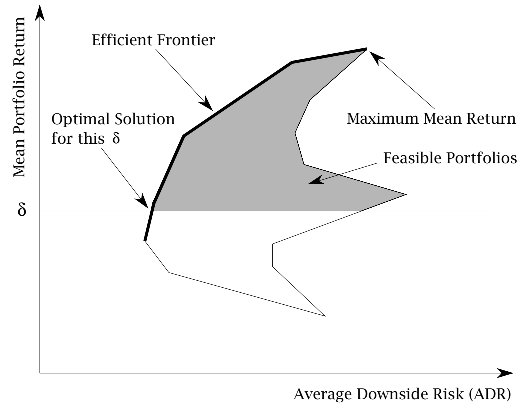
**Lecture 7**

- Portfolio Optimization – II
- GMS Stock Hedging
- Introduction to Retailer Simulation
- Summary and Preparation for next class

Note: Please bring your notebook computer to the next class (lecture 8).



## Portfolio Optimization Model




$$\min_{x_j} \quad ADR$$

subject to:

| | |
|---|---|
| (Average return) | $r_P \geq \delta$ |
| (Budget) | $\sum_{j=1}^{n} x_j = 1$ |
| (No short sales) | $x_j \geq 0$ for all $j$ |

## Spreadsheet Solution

Objective Function
=AVERAGE(A13:A16)

Decision Variables

=AVERAGE(D13:D16)

| A | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | INVESTLP.XLS | | Investment Linear Program | | | | | |
| 2 | | | | | | | | |
| 3 | | | | | | sum of | budget | |
| 4 | | | | Avg. return | | port wts | constraint | |
| 5 | ADR | | | r(P) | | 1.000 | = | 1 |
| 6 | 0.192 | | | 1.400 | | | | |
| 7 | | | | >= | | portfolio weights x(j) | | |
| 8 | | | Min return: | 1.4 | | 0.000 | 0.667 | 0.333 |
| 9 | | | | | | | | |
| 10 | downside | | | Return | | Scenario returns r(i,j) | | |
| 11 | return | | Constraint | by scen | | Securities | | |
| 12 | d(i) | d(i)+r(i) | d(i)+r(i)>=0 | r(i) | Scenario | 1 | 2 | 3 |
| 13 | 0.000 | 4.053 | >= | 4.053 | 1 | 5.51 | 4.80 | 2.56 |
| 14 | 0.000 | 0.460 | >= | 0.460 | 2 | -1.24 | 0.61 | 0.16 |
| 15 | 0.000 | 1.853 | >= | 1.853 | 3 | 5.46 | 3.60 | -1.64 |
| 16 | 0.767 | -0.000 | >= | -0.767 | 4 | -1.70 | -1.30 | 0.30 |

=SUMPRODUCT($F$8:$H$8,F13:H13)

For $\delta = 1.4$, the optimal solution is:

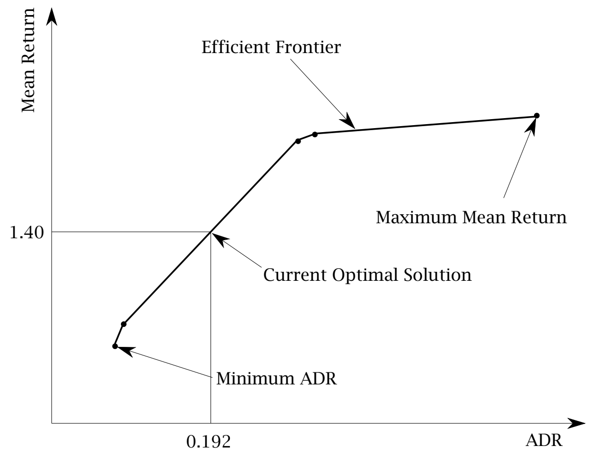
$x_1 = 0.000 \quad x_2 = 0.667 \quad x_3 = 0.333$
$r_1 = 4.053 \quad r_2 = 0.460 \quad r_3 = 1.853 \quad r_4 = -0.767$
$d_1 = 0.000 \quad d_2 = 0.000 \quad d_3 = 0.000 \quad d_4 = 0.767$

with $ADR = 0.192$ and $r_P = 1.400$ (all returns expressed in percent).



## Efficient Frontier

As $\delta$ is varied, the optimal solutions to the LP trace out the *efficient frontier*.



## Sensitivity Analysis

If $\delta$ is increased from 1.4 to 1.5, i.e., if the required minimum average portfolio return is increased, what will the new $ADR$ be? Answer: Check the dual price of the "Min. $r_P$" constraint.
The dual price of this constraint is 0.253. Recall that

$$\text{Dual price } = \frac{\Delta ADR}{\Delta r_P}.$$

The change in $ADR$ is

$$\begin{aligned}\Delta ADR &= \text{Dual Price } \times \Delta r_P \\ &= 0.253 \times 0.1 = 0.0253.\end{aligned}$$

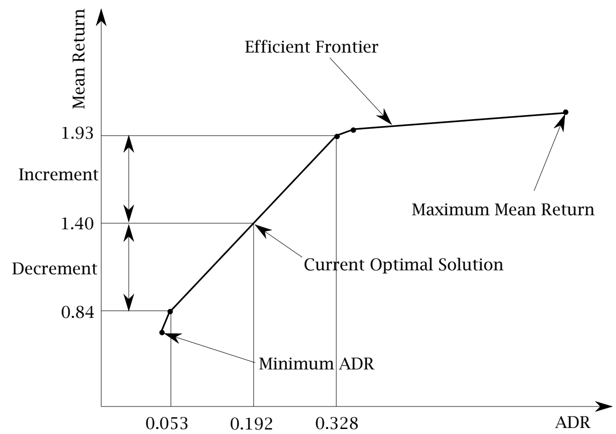
So for $\delta = 1.5$, the minimum $ADR$ is

$$\begin{aligned}\text{New } ADR &= \text{Original } ADR + \Delta ADR \\ &= 0.192 + 0.025 = 0.217.\end{aligned}$$

The dual price of the "Min. $r_P$" constraint gives information about the slope of the efficient frontier. Because of the way efficient frontiers are typically graphed (with $r_P$ on the vertical axis and $ADR$ on the horizontal axis), the dual price gives the inverse of the slope of the efficient frontier.



## Efficient Frontier




## Righthand Side Ranges

The breakpoints of the efficient frontier are given by the "Increment" and "Decrement" values for the "Min. $r_P$" constraint. For $\delta = 1.4$, the dual price is 0.253, the "increment" is 0.53 and the "decrement" is 0.56. Hence, the breakpoints on the efficient frontier occur at $(ADR, r_P) = (0.328, 1.93)$ and $(ADR, r_P) = (0.053, 0.84)$.

## Spreadsheet Solution and Sensitivity Report

| A | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | INVESTLP.XLS | | Investment Linear Program | | | | | |
| 2 | | | | | | | | |
| 3 | | | | | | sum of | budget | |
| 4 | | | | Avg. return | | port wts | constraint | |
| 5 | ADR | | | r(P) | | 1.000 | = | 1 |
| 6 | 0.192 | | | 1.400 | | | | |
| 7 | | | | >= | | portfolio weights x(j) | | |
| 8 | | | Min return: | 1.4 | | 0.000 | 0.667 | 0.333 |
| 9 | | | | | | | | |
| 10 | downside | | | Return | | Scenario returns r(i,j) | | |
| 11 | return | | Constraint | by scen | | Securities | | |
| 12 | d(i) | d(i)+r(i) | d(i)+r(i)>=0 | r(i) | Scenario | 1 | 2 | 3 |
| 13 | 0.000 | 4.053 | >= | 4.053 | 1 | 5.51 | 4.80 | 2.56 |
| 14 | 0.000 | 0.460 | >= | 0.460 | 2 | -1.24 | 0.61 | 0.16 |
| 15 | 0.000 | 1.853 | >= | 1.853 | 3 | 5.46 | 3.60 | -1.64 |
| 16 | 0.767 | -0.000 | >= | -0.767 | 4 | -1.70 | -1.30 | 0.30 |

Changing Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $F$8 | Min return: portfolio weights x(j) | 0.000 | 0.080 | 0 | 1E+30 | 0.079778831 |
| $G$8 | Min return: constraint | 0.667 | 0.000 | 0 | 0.07593985 | 0.4 |
| $H$8 | Min return: | 0.333 | 0.000 | 0 | 0.4 | 1.578125 |
| $A$13 | d(i) | 0.000 | 0.250 | 0.25 | 1E+30 | 0.25 |
| $A$14 | d(i) | 0.000 | 0.250 | 0.25 | 1E+30 | 0.25 |
| $A$15 | d(i) | 0.000 | 0.250 | 0.25 | 1E+30 | 0.25 |
| $A$16 | d(i) | 0.767 | 0.000 | 0.25 | 1E+30 | 0.25 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$13 | d(i)+r(i) | 4.053 | 0.000 | 0 | 4.053333333 | 1E+30 |
| $B$14 | d(i)+r(i) | 0.460 | 0.000 | 0 | 0.46 | 1E+30 |
| $B$15 | d(i)+r(i) | 1.853 | 0.000 | 0 | 1.853333333 | 1E+30 |
| $B$16 | d(i)+r(i) | 0.000 | 0.250 | 0 | 1E+30 | 0.766666667 |
| $F$5 | r(P) port wts | 1.000 | -0.162 | 1 | 0.666098885 | 0.273670558 |
| $D$6 | r(P) | 1.400 | 0.253 | 1.4 | 0.5275 | 0.55971374 |

The report shows the dual (shadow) price of the "Min. $r_P$" constraint in the row $D$6. The dual price is 0.253, the increment is 0.528 and the decrement is 0.560.



## GMS Stock Hedging

- Gold mining stock (GMS) is identified as an attractive investment
  - → New mining equipment
  - → New land mining rights
  - → Gold is a safe haven if there is a global monetary crisis
  - → Supply and demand favor gold price increase
- Potential problem areas
  - → GMS is a highly leveraged company
  - → Investment in GMS alone is highly risky
  - → Gold prices are not sure to rise
  - → LionFund is a conservative risk-averse fund

How to participate in the upside potential of GMS stock without incurring the risk of this investment?

## GMS Stock Hedging

**Table 1.** Scenarios and Probabilities for GMS Stock in One Month

| Scenario | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Probability | 0.05 | 0.10 | 0.20 | 0.30 | 0.20 | 0.10 | 0.05 |
| GMS Price | 150 | 130 | 110 | 100 | 90 | 80 | 70 |

**Table 2.** Put Option Prices (Today)

| Put option | A | B | C |
|---|---|---|---|
| Strike price | 90 | 100 | 110 |
| Option price | \$2.20 | \$6.40 | \$12.50 |

*Problem:* What is the minimum risk (i.e., minimum standard deviation) portfolio that invests all \$10 million in stock and options?

We first need to compute the returns of each security in each of the scenarios.



## Scenario Returns

Suppose scenario 7 occurs. What is the return of GMS stock? What is the return of put option C?

If there are no intermediate cash flows, the return of a security is

$$\text{Return} = \frac{\text{Final price} - \text{Initial price}}{\text{Initial price}}.$$

For GMS stock in scenario 7, this gives

$$-30\% = \frac{70 - 100}{100}.$$

The final value of a put option is given by

$$\max(K - S, 0),$$

where $S$ is the stock price at the option expiration and $K$ is the option strike price.

For put option C, its final value in scenario 7 is

$$40 = \max(110 - 70, 0).$$

Hence, the return of put option C if scenario 7 occurs is

$$220.0\% = \frac{40 - 12.50}{12.50}.$$

## Scenario Returns (continued)

| B | F | G | H | I | J |
|---|---|---|---|---|---|
| 4 | | Gold | Put | Put | Put |
| 5 | | Stock | Option A | Option B | Option C |
| 6 | Initial Price | 100 | 2.20 | 6.40 | 12.85 |
| 7 | | | | | |
| 8 | Option strike price | | 90 | 100 | 110 |
| 9 | | | | | |
| 10 | Table of Final Prices by Scenario | | | | |
| 11 | | Gold | Put | Put | Put |
| 12 | Scenario | Stock | Option A | Option B | Option C |
| 13 | 1 | 150 | 0 | 0 | 0 |
| 14 | 2 | 130 | 0 | 0 | 0 |
| 15 | 3 | 110 | 0 | 0 | 0 |
| 16 | 4 | 100 | 0 | 0 | 10 |
| 17 | 5 | 90 | 0 | 10 | 20 |
| 18 | 6 | 80 | 10 | 20 | 30 |
| 19 | 7 | 70 | 20 | 30 | 40 |

=MAX(J$8-$G19,0)
(copied to the range B!H13:B!J19)

| A | F | G | H | I | J |
|---|---|---|---|---|---|
| 10 | Scenario returns (in percent) | | | | |
| 11 | | Gold | Put | Put | Put |
| 12 | Scen | Stock | Option A | Option B | Option C |
| 13 | 1 | 50.0 | -100.0 | -100.0 | -100.0 |
| 14 | 2 | 30.0 | -100.0 | -100.0 | -100.0 |
| 15 | 3 | 10.0 | -100.0 | -100.0 | -100.0 |
| 16 | 4 | 0.0 | -100.0 | -100.0 | -20.0 |
| 17 | 5 | -10.0 | -100.0 | 56.2 | 60.0 |
| 18 | 6 | -20.0 | 354.5 | 212.5 | 140.0 |
| 19 | 7 | -30.0 | 809.1 | 368.8 | 220.0 |

100*(B!J19–B!J$6)/B!J$6
(copied to the range A!G13:A!J19)



## GMS Hedging Spreadsheet Model

=SUMPRODUCT(D13:D19,E13:E19)

Objective function
=SQRT(SUMPRODUCT(B13:B19,E13:E19))

+J6*1E7/B!J6

| A | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | GOLD.XLS | | Gold Stock Hedging | | | | sum of | budget | | |
| 2 | | | | | | | port wts | constraint | | |
| 3 | | | | Avg. return | | | 1.000 | = | 1 | |
| 4 | | STD | | (in percent) | | | | | | |
| 5 | | 7.95 | | 1.095 | | | portfolio weights | | | |
| 6 | | | | | | | 0.849 | 0.000 | 0.000 | 0.151 |
| 7 | | | | | | | number of units | | | |
| 8 | | | | | | | 84,913 | 0 | 0 | 120,694 |
| 9 | | | | | | | | | | |
| 10 | | | | Portfolio | | | Scenario returns (in percent) | | | |
| 11 | | | | return | | | Gold | Put | Put | Put |
| 12 | | (r(i)-av.ret)^2 | | r(i) | Prob | Scen | Stock | Option A | Option B | Option C |
| 13 | | 690.38 | | 27.37 | 0.05 | 1 | 50.0 | -100.0 | -100.0 | -100.0 |
| 14 | | 86.35 | | 10.39 | 0.10 | 2 | 30.0 | -100.0 | -100.0 | -100.0 |
| 15 | | 59.14 | | -6.60 | 0.20 | 3 | 10.0 | -100.0 | -100.0 | -100.0 |
| 16 | | 16.91 | | -3.02 | 0.30 | 4 | 0.0 | -100.0 | -100.0 | -20.0 |
| 17 | | 0.29 | | 0.56 | 0.20 | 5 | -10.0 | -100.0 | 56.2 | 60.0 |
| 18 | | 9.27 | | 4.14 | 0.10 | 6 | -20.0 | 354.5 | 212.5 | 140.0 |
| 19 | | 43.85 | | 7.72 | 0.05 | 7 | -30.0 | 809.1 | 368.8 | 220.0 |

=SUMPRODUCT($G$6:$J$6,G19:J19)

- The objective is to minimize standard deviation.
- The optimal solution is to have 84.9% of the portfolio in gold stock and 15.1% in put option C.
- With a $10 million budget, this implies purchasing 84,913 shares of stock and 120,694 C puts.

## GMS Hedging without Nonnegativity

| A | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | GOLD.XLS | | Gold Stock Hedging | | | | sum of | budget | | |
| 2 | | | | | | | port wts | constraint | | |
| 3 | | | | Avg. return | | | 1.000 | = 1 | | |
| 4 | | STD | | (in percent) | | | | | | |
| 5 | | 7.18 | | 1.651 | | | portfolio weights | | | |
| 6 | | | | | | | 0.830 | -0.001 | -0.066 | 0.238 |
| 7 | | | | | | | number of units | | | |
| 8 | | | | | | | 82,972 | (3,797) | (103,844) | 190,057 |
| 9 | | | | | | | | | | |
| 10 | | | | Portfolio | | | Scenario returns (in percent) | | | |
| 11 | | | | return | | | Gold | Put | Put | Put |
| 12 | | (r(i)-av.ret)^2 | | r(i) | Prob | Scen | Stock | Option A | Option B | Option C |
| 13 | | 520.17 | | 24.46 | 0.05 | 1 | 50.0 | -100.0 | -100.0 | -100.0 |
| 14 | | 38.60 | | 7.86 | 0.10 | 2 | 30.0 | -100.0 | -100.0 | -100.0 |
| 15 | | 107.78 | | -8.73 | 0.20 | 3 | 10.0 | -100.0 | -100.0 | -100.0 |
| 16 | | 0.11 | | 1.98 | 0.30 | 4 | 0.0 | -100.0 | -100.0 | -20.0 |
| 17 | | 0.42 | | 2.30 | 0.20 | 5 | -10.0 | -100.0 | 56.2 | 60.0 |
| 18 | | 0.35 | | 2.25 | 0.10 | 6 | -20.0 | 354.5 | 212.5 | 140.0 |
| 19 | | 0.29 | | 2.19 | 0.05 | 7 | -30.0 | 809.1 | 368.8 | 220.0 |

- The nonnegativity constraint on portfolio weights is removed to allow short sales.
- The objective is to minimize standard deviation.
- The optimal solution is to have 83.0% of the portfolio in gold stock, short 0.1% of put A, short 6.6% of put B, and have 23.8% in put C.
- With a $10 million budget, this implies purchasing 82,972 shares of stock, shorting 3,797 A puts, shorting 103,844 B puts, and purchasing 190,057 C puts.



## GMS Hedging with ADR Objective

| A | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | GOLD.XLS | | Gold Stock Hedging | | | | sum of | budget | | |
| 2 | | | | | | | port wts | constraint | | |
| 3 | | | | Avg. return | | | 1.000 | = 1 | | |
| 4 | ADR | | | (in percent) | | | | | | |
| 5 | 1.10 | | | 1.509 | | | portfolio weights | | | |
| 6 | | | | | | | 0.859 | -0.000 | -0.035 | 0.176 |
| 7 | | | | | | | number of units | | | |
| 8 | | | | | | | 85,903 | (0) | (55,066) | 140,969 |
| 9 | | | | | | | | | | |
| 10 | downside | | | Portfolio | | | Scenario returns (in percent) | | | |
| 11 | return | | d(i)+r(i) | return | | | Gold | Put | Put | Put |
| 12 | d(i) | d(i)+r(i) | >= 0? | r(i) | Prob | Scen | Stock | Option A | Option B | Option C |
| 13 | 0.00 | 28.85 | >= 0 | 28.85 | 0.05 | 1 | 50.0 | -100.0 | -100.0 | -100.0 |
| 14 | 0.00 | 11.67 | >= 0 | 11.67 | 0.10 | 2 | 30.0 | -100.0 | -100.0 | -100.0 |
| 15 | 5.51 | 0.00 | >= 0 | -5.51 | 0.20 | 3 | 10.0 | -100.0 | -100.0 | -100.0 |
| 16 | 0.00 | -0.00 | >= 0 | -0.00 | 0.30 | 4 | 0.0 | -100.0 | -100.0 | -20.0 |
| 17 | 0.00 | -0.00 | >= 0 | -0.00 | 0.20 | 5 | -10.0 | -100.0 | 56.2 | 60.0 |
| 18 | 0.00 | -0.00 | >= 0 | -0.00 | 0.10 | 6 | -20.0 | 354.5 | 212.5 | 140.0 |
| 19 | 0.00 | -0.00 | >= 0 | -0.00 | 0.05 | 7 | -30.0 | 809.1 | 368.8 | 220.0 |

- The nonnegativity constraint on portfolio weights is removed to allow short sales.
- The objective is to minimize ADR.
- The optimal solution is to have 85.9% of the portfolio in gold stock, short 3.5% of put B, and have 17.6% in put C.
- With a $10 million budget, this implies purchasing 85,903 shares of stock, shorting 55,066 B puts, and purchasing 140,969 C puts.

## Comparison of Alternative Solutions

Scenario Returns for Different Portfolios

| Scen | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Prob | 0.05 | 0.10 | 0.20 | 0.30 | 0.20 | 0.10 | 0.05 |
| Port 1 | 50.0 | 30.0 | 10.0 | 0.0 | −10.0 | −20.0 | −30.0 |
| Port 2 | 46.8 | 27.2 | 7.6 | −2.2 | −11.9 | −11.9 | −11.9 |
| Port 3 | 27.4 | 13.4 | −6.6 | −3.0 | 0.6 | 4.1 | 7.7 |
| Port 4 | 24.5 | 7.9 | −8.7 | 2.0 | 2.3 | 2.3 | 2.2 |
| Port 5 | 28.9 | 11.7 | −5.5 | 0.0 | 0.0 | 0.0 | 0.0 |

Portfolio 1: 100% in gold stock
Portfolio 2: 97.8% in stock, 2.2% in put option A (97,847 shares and 97,847 options)
Portfolio 3: 84.9% in stock, 15.1% in put option C
Portfolio 4: 83.0% in stock, −0.1% in put A, −6.6% in put B, and 23.8% in put option C
Portfolio 5: 85.9% in stock, −3.5% in put B, and 17.6% in put option C

Portfolio 1: avg ret = 2.00%, std = 18.3%, ADR = 5.5%
Portfolio 2: avg ret = 1.76%, std = 15.6%, ADR = 4.8%
Portfolio 3: avg ret = 1.10%, std = 8.0%, ADR = 2.2%
Portfolio 4: avg ret = 1.65%, std = 7.2%, ADR = 1.8%
Portfolio 5: avg ret = 1.51%, std = 7.7%, ADR = 1.1%



## GMS Hedging Summary

- Portfolio 1: Investment in GMS stock alone
  - → This investment is quite risky
  - → STD = 18.3%, ADR = 5.5%, potential loss of 30%
- Portfolio 2: Hedging each share of stock with one put option A
  - → Reduces risk only slightly
- Portfolio 3: Minimum variance solution with nonnegative portfolio weights
  - → Reduces risk significantly
- Portfolio 4: Minimum variance solution with negative portfolio weights allowed
  - → Reduces risk and increase average return compared to portfolio 3
- Portfolio 5: Minimum *ADR* solution with negative portfolio weights allowed
  - → Maximum loss only 5.5%. Better than portfolio 4?

## Portfolio Optimization Software

Many companies sell software packages for portfolio optimization. A few examples include:

- BARRA
- Sponsor-Software Systems, Inc.
  → The Asset Allocation Expert (AAE)
- Wilson Associates
  → Capital Asset Management System (CAMS)
- LaPorte
  → LaPorte Asset Allocation System

Typical features of these systems include:

- Historical databases
- Graphical capabilities
- Reporting capabilities
- Technical support

Typical prices are \$2,000 – \$10,000 for an initial license plus \$1,000 – \$4,000 per year for upgrades and database updates.



## Other Applications

This portfolio optimization model is one example of a *scenario LP* or *stochastic LP*. Similar models have been developed for:

- Bond portfolio selection
  → scenarios are future yield curve changes
  → SEC now regulates S&L's based on minimum capital requirements based on a range of future yield curve scenarios (typically parallel yield curve shifts)
- Corporate risk management
  → scenarios represent corporate risk factors

A model similar to the GMS case was developed last fall by Cort Gwon (Columbia MBA '95):

- LibertyView Capital Management
- Invests in undervalued high yield (junk) bonds
- Spreadsheet optimization model is now used to hedge bond investments using stock and options
- Scenarios developed by the traders

## Introduction to Retailer Simulation

*Retailer* is a simulation exercise that places the user in the role of a manager of a large chain of retail clothing stores. In this setting, yield management boils down to deciding the *timing* and *magnitude* of price reductions.

Background Information:

Fashion Retail Merchandise

- Staple Items
  - → Regularly purchased items, e.g., socks, underwear, T-shirts, etc.
- Fashion Items
  - → Items with a strong fashion component; quick obsolescence
  - → Specific selling seasons, e.g., winter, spring, cruise, holiday
  - → Define the "style" of a store and position it relative to competitors
  - → Demand is highly erratic: "hit" items can sell out in a few weeks, other items ("crawlers" or "dogs") can sell very slowly



## Production and Distribution

- Garment design
  - → Creative process, most important phase
  - → Basic silhouettes, colors, and fabrics chosen
  - → Typically begins *one year in advance* of the target selling season
- Production qty decision, material procurement
  - → Based on rough forecasts of likely sales
  - → Vagaries of fashion and long lead times often result in highly inaccurate forecasts
  - → Procurement lead time: 1–2 weeks for standard in-stock fabrics to several months for special-order fabrics
- Garment assembly
  - → In-house or through subcontractors
  - → Lead time: under 4 weeks (in-house) to several months (e.g., overseas subcontractor)
- Distribution
  - → Takes 1–2 weeks (domestic supplier) to 4–6 weeks (e.g., overseas supplier using container ships for transportation)

## Retailer Background

- Procurement and production lead time
  - → Long for fashion items: ranging from many weeks to several months
  - → Fashion items are usually produced in a *single production run*
  - → No opportunity for restocking during a short 8–15 week selling season.
- Matching supply and demand to maximize revenue
  - → Transfer merchandise between stores
  - → Price changes: timing and magnitude decisions
- POS technology
  - → Links cash registers to home office computer
  - → Links distribution centers to home office computer
  - → Managers have a "real-time" view of sales and inventory throughout the distribution chain



## Financial Implications

The GAP – Operating Statement Information

| ($ Millions) | 1991 | 1992 |
|---|---|---|
| Net Sales | $2,518.0 | $2,960.0 |
| Cost of Goods Sold | 1,568.0 | 1,955.6 |
| S,G & A | 575.7 | 661.3 |
| Interest Expense | 3.5 | 3.8 |
| Pretax Income | 370.8 | 339.8 |
| Taxes | 140.9 | 129.1 |
| Net Income | 229.9 | 210.7 |
| | | |
| EPS | $1.62 | $1.47 |
| Shares Out (mil) | 142.0 | 143.7 |
| Sales % Change | 30.3% | 17.7% |
| Comp-Stores | 13.0 | 5.0 |
| | | |
| % OF SALES | | |
| Cost of Goods Sold | 62.3% | 66.1% |
| S,G & A | 22.9 | 22.3 |
| Interest Expense | 0.1 | 0.1 |
| Pretax Income | 14.7 | 11.5 |
| Tax Rate | 38.0 | 38.0 |

Suppose a better markdown strategy produced a 2% revenue increase in 1992:

$\Longrightarrow$ \$59 million increase in sales

$\Longrightarrow$ No change in cost of goods sold

$\Longrightarrow$ 17% increase in pretax income and net income

$\Longrightarrow$ 17% increase in earnings per share

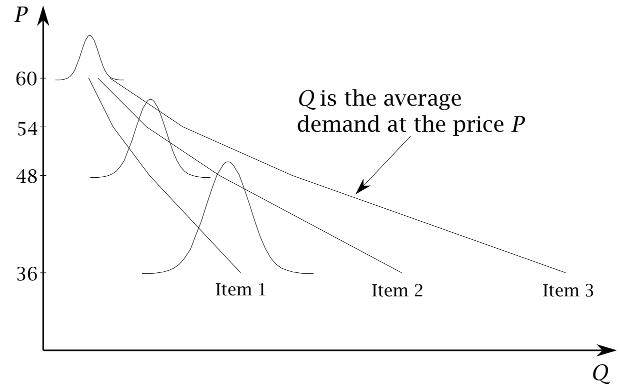
Relatively small changes in revenue can have a substantial impact on a company's bottom line.

## Retailer Parameters

- Stores are stocked with 2,000 units of a single fashion item
  - → Management hopes for strong sales but demand is hard to predict
  - → No chance for restocking the item or reallocating among stores
- Initial price is \$60
- 15 week selling season
- Goal: *maximize the revenue* from the 2,000 units
  - → Production and distribution costs have already been paid; they are sunk costs
- Four allowable price levels
  - → \$60 (full price), \$54 (10% off), \$48 (20% off), \$36 (40% off)
- Management policy: price cannot be raised once it has been cut
- All items in stores that are not sold at the end of 15 weeks are sold to discounters ("jobbers") for \$25 per unit (salvage value)



## Retailer Demand Curves




- There is a different demand curve for each item
- For a given item, demand is random from week to week (even at the same price)
- The demand curve for each item is unknown (i.e., at the beginning of each season, it is not known whether the item is more like Item 1 or Item 3.)

## Preliminary Analysis

Problem: How to develop a sensible pricing policy?

### Historical Sales Data

- Historical data on 15 previous fashion items is stored in the spreadsheet `RETAIL.XLS`.
- Each item is different — some turned out to be fast sellers while others did not sell so well.
- Although the items were different, their responsiveness to price cuts was quite similar.
- "Deseasonalized" data: the data has been normalized to remove the predictable effects of seasons and holidays on sales figures. (These effects are also removed from the *Retailer* simulation exercise.)
- Sales are quite variable: even at the same price, sales can vary considerably from week to week due to weather, competitors, and a host of other factors.



| A | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | RETAIL.XLS | | | | | |
| 2 | | | | | | |
| 3 | | Historical sales data for 15 different items | | | | |
| 4 | | for use with the RETAILER simulation game. | | | | |
| 5 | | | | | | |
| 6 | | | Qty on | | | |
| 7 | Item | Week | hand | Price | Sales | |
| 8 | 1 | 1 | 2000 | 60 | 57 | |
| 9 | | 2 | 1943 | 60 | 98 | |
| 10 | | 3 | 1845 | 60 | 55 | |
| 11 | | 4 | 1790 | 60 | 41 | |
| 12 | | 5 | 1749 | 60 | 60 | |
| 13 | | 6 | 1689 | 60 | 39 | |
| 14 | | 7 | 1650 | 54 | 106 | |
| 15 | | 8 | 1544 | 54 | 55 | |
| 16 | | 9 | 1489 | 54 | 64 | |
| 17 | | 10 | 1425 | 54 | 43 | |
| 18 | | 11 | 1382 | 54 | 131 | |
| 19 | | 12 | 1251 | 54 | 112 | |
| 20 | | 13 | 1139 | 54 | 62 | |
| 21 | | 14 | 1077 | 54 | 31 | |
| 22 | | 15 | 1046 | 54 | 80 | |
| 23 | | 16 | 966 | | | |
| 24 | | | | | | |
| 25 | 2 | 1 | 2000 | 60 | 115 | |
| 26 | | 2 | 1885 | 60 | 105 | |
| 27 | | 3 | 1780 | 60 | 136 | |
| 28 | | 4 | 1644 | 60 | 115 | |
| 29 | | 5 | 1529 | 60 | 73 | |
| 30 | | 6 | 1456 | 60 | 102 | |
| 31 | | 7 | 1354 | 54 | 58 | |
| 32 | | 8 | 1296 | 54 | 187 | |
| 33 | | 9 | 1109 | 54 | 198 | |
| 34 | | 10 | 911 | 54 | 196 | |
| 35 | | 11 | 715 | 54 | 132 | |
| 36 | | 12 | 583 | 54 | 60 | |
| 37 | | 13 | 523 | 54 | 119 | |
| 38 | | 14 | 404 | 54 | 131 | |
| 39 | | 15 | 273 | 54 | 215 | |
| 40 | | 16 | 58 | | | |
| 41 | | | | | | |
| 42 | 3 | 1 | 2000 | 60 | 75 | |
| 43 | | 2 | 1925 | 60 | 82 | |
| 44 | | 3 | 1843 | 60 | 63 | |
| 45 | | 4 | 1780 | 60 | 53 | |
| 46 | | 5 | 1727 | 60 | 63 | |
| 47 | | 6 | 1664 | 60 | 20 | |
| 48 | | 7 | 1644 | 54 | 57 | |
| 49 | | 8 | 1587 | 54 | 118 | |
| 50 | | 9 | 1469 | 54 | 90 | |
| 51 | | 10 | 1379 | 54 | 51 | |
| 52 | | 11 | 1328 | 54 | 126 | |
| 53 | | 12 | 1202 | 54 | 73 | |
| 54 | | 13 | 1129 | 54 | 88 | |
| 55 | | 14 | 1041 | 54 | 64 | |
| 56 | | 15 | 977 | 54 | 74 | |
| 57 | | 16 | 903 | | | |
| 58 | | | | | | |

## Preliminary Analysis (continued)

In your group, analyze the historical data in `RETAIL.XLS` and try to develop a sensible markdown strategy. In your analysis, you might want to answer:

- What is the average effect on sales of each size price cut? For example, for a price cut from \$60 to \$54, what is the average increase in weekly sales?
- How variable are sales from one item to the next?

In developing a strategy, you might want to consider:

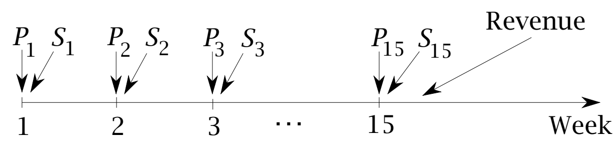
- If demand was not variable, what would be the optimal price cut strategy? For example, suppose the demand at a price of \$60 was a constant 80 items per week. Using your estimated demand sensitivities, to what level and at what point in the selling season would you cut the price?
- How might your strategy be altered to account for uncertainty in demand?

You should work out any desired formulas in advance, so that necessary calculations can be done simply and quickly in class.



## Retailer

*Retailer* is a multiperiod simulation.




$P_i$ is the price set for week $i$ (decision variable)
$S_i$ is the sales in week $i$ (random).

The *Retailer* simulation will do some calculations automatically.

## Retailer Simulation Screen

| Week | Qty on hand | Price | Sales | Rev | Cum Rev | Avg Sales | Std Err | Proj Sales |
|---|---|---|---|---|---|---|---|---|
| 1 | 2000 | 60 | 99 | 5940 | 5940 | 99 | – | 1485 |
| 2 | 1901 | | | | | | | |

Columns labeled Week, Qty on hand, Price, and Sales are self-explanatory.

Rev: The revenue for the current week, i.e.,

$$\text{Rev} = \text{Price} \times \text{Sales} .$$

Cum Rev: Total (or cumulative) revenue since the beginning of the selling season.

Avg Sales: The average of weekly sales at the current price.

Std Err: Standard error of the average sales, i.e., $s/\sqrt{n}$ where $s$ is the std dev of sales and $n$ is the number of weeks of sales (at the current price).

Proj Sales: Projected total sales after 15 weeks. The projection is made using cumulative sales thus far plus sales continuing at the current average. For example, $1485 = 99 \times 15$.



## Retailer Simulation Screen (continued)

| Week | Qty on hand | Price | Sales | Rev | Cum Rev | Avg Sales | Std Err | Proj Sales |
|---|---|---|---|---|---|---|---|---|
| 1 | 2000 | 60 | 99 | 5940 | 5940 | 99 | – | 1485 |
| 2 | 1901 | 60 | 53 | 3180 | 9120 | 76 | 23 | 1140 |
| 3 | 1848 | | | | | | | |

The user had the choice of four price levels: \$60, \$54, \$48, and \$36. The user chose to maintain the price at \$60.

Cum Rev: \$9120 = 5940 + 3180.

Avg Sales: 76 = (99 + 53) / 2.

Std Err: $23 = s/\sqrt{2}$, where $s = 32.5$.

Proj Sales: Current total sales + future sales at average rate:

$$1140 = (99 + 53) + 13 \times 76.$$

At this point, the user can again choose from 4 price levels: \$60, \$54, \$48, and \$36. The user chose to cut the price to \$54.

## Retailer Simulation Screen (continued)

| Week | Qty on hand | Price | Sales | Rev | Cum Rev | Avg Sales | Std Err | Proj Sales |
|---|---|---|---|---|---|---|---|---|
| 1 | 2000 | 60 | 99 | 5940 | 5940 | 99 | – | 1485 |
| 2 | 1901 | 60 | 53 | 3180 | 9120 | 76 | 23 | 1140 |
| 3 | 1848 | 54 | 85 | 4590 | 13710 | 85 | – | 1257 |
| 4 | 1763 | | | | | | | |

Cum Rev: $13710 = 5940 + 3180 + 4590.

Avg Sales: 85 (average at the current price of $54).

Std Err: Undefined, since there is only one week of sales at the current price of $54.

Proj Sales: Current total sales + future sales at average rate:

$$1257 = (99 + 53 + 85) + 12 \times 85.$$

At this point, the user can choose from only 3 price levels: $54, $48, and $36.

At the end of 15 weeks, *revenue from sales will be added to revenue from salvage to determine total revenue.*



## Summary

- Linear and nonlinear formulations of the portfolio optimization model
- Interpretation of dual price information
- Interpretation of righthand side range information (e.g., dual price “increment” and “decrement”)
- Application to stock hedging using options

## For next class

- Quiz is due next lecture. This is an individual assignment.
- Please remember to bring your notebook computer to the next class.
- Read the case "Retailer: A Retail Pricing Simulation Exercise" on pp.529–534 in the W&A text. Download the Retailer files from the course webpage at `http://www.columbia.edu/cu/business/courses/`. (Please put all of the Retailer-related files into a directory `C:\RETAIL` on your computer.)
- Optional readings: "His Goal: No Room at the Inns," "Computers as Price Setters Complicate Travelers' Lives," "Making Supply Meet Demand in an Uncertain World," and "Yield Management at American Airlines" in the readings book.