COLUMBIA
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Lecture 6

Portfolio Optimization

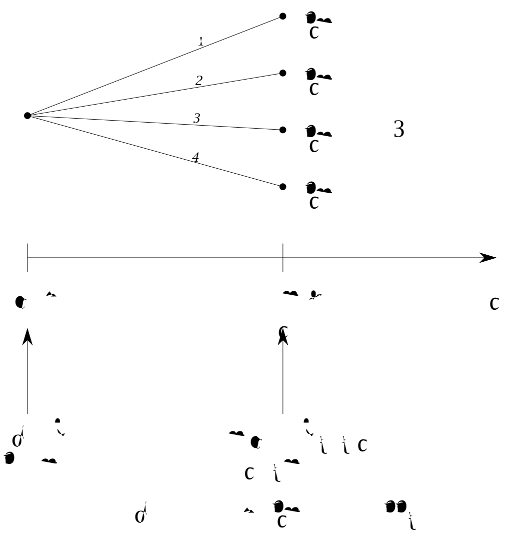
Problem: [illegible]

Example. [illegible]

Definition: [illegible] x_j, $j = 1, \ldots, n$ [illegible] $\sum_{j=1}^{n} x_j = 1$ [illegible] x_j [illegible] $j = 1, \ldots, n$ [illegible]

A Model of the Uncertain Future

[illegible]



[illegible]

Definition: [illegible]

Scenario Returns and Probabilities

Table. [illegible]

[illegible]

[illegible] r_{ij} [illegible] j [illegible] i [illegible] r_3 f 3:[illegible]%

[illegible]

Portfolio Returns

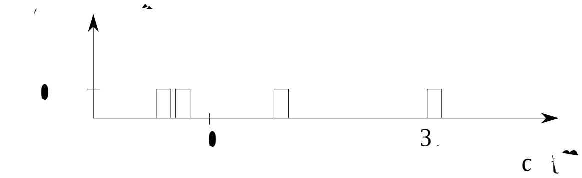
[illegible] i [illegible] „x ; : : : ; x_n… [illegible] i [illegible] r_i

$$r_i \; f \; \sum_{j f}^{n} r_{ij} x_j :$$

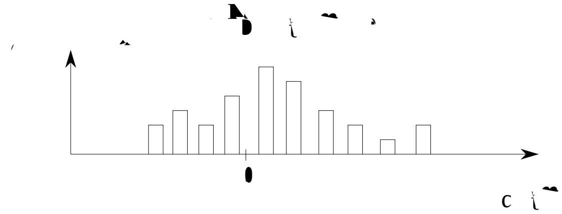
„ …

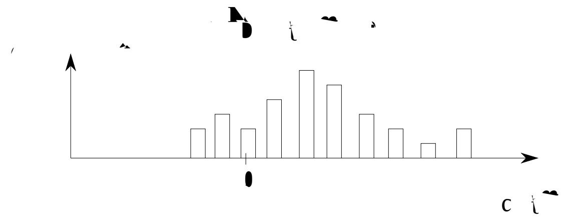
Portfolio Returns (continued)

Example.



Preferences for Return Distributions



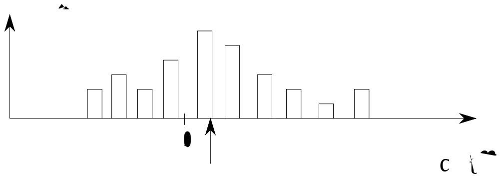


Average Portfolio Return

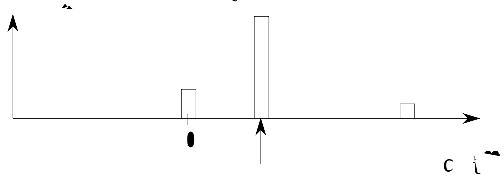
Definition:

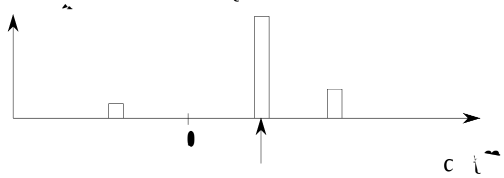
r_P

$$r_P \; f \; \sum_{i f}^{m} p_i r_i :$$



Preferences for Return Distributions (continued)





Average Downside Risk

Average Downside Risk (continued)

Average Downside Risk (continued)

r_i ... i ... i

$$d_i f \begin{cases} & r_i > \\ r_i & r_i \end{cases} \quad „3…$$

Definition: a a ... ADR

$$ADR\ f \sum_{i f}^{m} p_i d_i : \quad „…$$

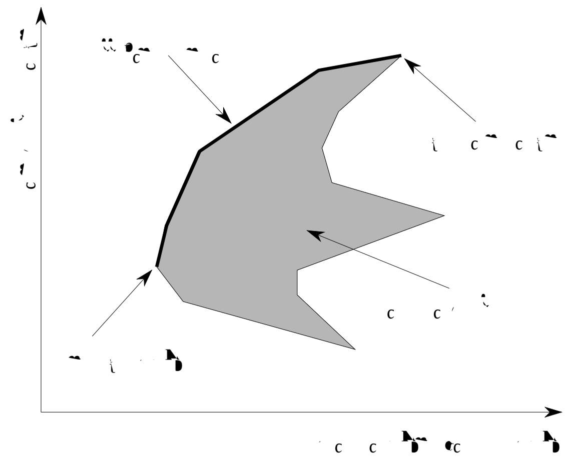
p_i ... i

r_P ... ADR ... r_P ... ADR

Efficient Frontier

$„x ; : : : ; x_n…$ $\sum_{j f}^{n} x_j\ f$

x_j ... r_P ... ADR



Portfolio Optimization Model

ADR ... r_P

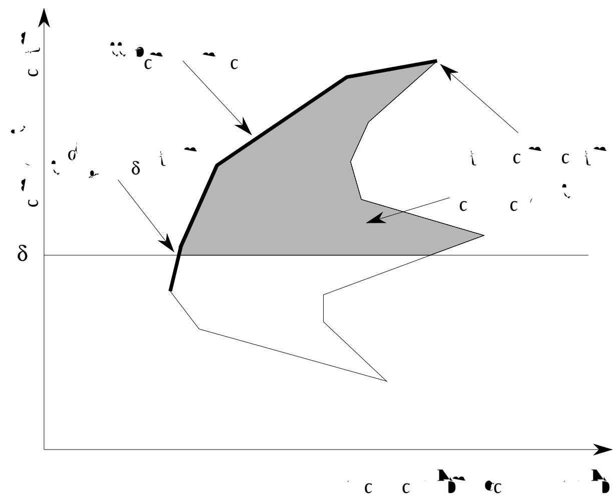
x_j ADR

r_P

$$\sum_{j\ f}^{n} x_j \ f$$

x_j ... j

Portfolio Optimization Model (continued)



Details of the Optimization Model

Table.

				3
3			3	
			3	3

x ; x ; x_3 ...

r f : x , : x , : x_3

r f : x , : x , : x_3

3 r_3 f : x , 3: x : x_3

r f : x :3 x , :3 x_3

r_P f = r , r , r_3 , r ...

ADR

$$d_i\ f \begin{cases} & r_i > \\ r_i & r_i \end{cases}$$

ADR f = d , d , d_3 , d ...

Downside Risk Defined

d_i

$$d_i\ f \begin{cases} & r_i > \\ r_i & r_i \end{cases}$$

d_i

d_i d_i r_i

r f

d d

ADR d

d f

r f 3

d d 3

d f

d_i

ADR

Mean-ADR Portfolio Optimization Model

ADR

Spreadsheet Solution

=AVERAGE(A13:A16)

=AVERAGE(D13:D16)

	A	B	C	D	E	F	G	H
1	INVESTLP.XLS		Investment Linear Program					
2								
3						sum of	budget	
4				Avg. return		port wts	constraint	
5	ADR			r(P)		1.000	=	1
6	0.192			1.400				
7				>=		portfolio weights x(j)		
8			Min return:	1.4		0.000	0.667	0.333
9								
10	downside			Return		Scenario returns r(i,j)		
11	return		Constraint	by scen		Securities		
12	d(i)	d(i)+r(i)	d(i)+r(i)>=0	r(i)	Scenario	1	2	3
13	0.000	4.053	>=	4.053	1	5.51	4.80	2.56
14	0.000	0.460	>=	0.460	2	-1.24	0.61	0.16
15	0.000	1.853	>=	1.853	3	5.46	3.60	-1.64
16	0.767	-0.000	>=	-0.767	4	-1.70	-1.30	0.30

=SUMPRODUCT($F$8:$H$8,F13:H13)

Optimization Model Results

[illegible]

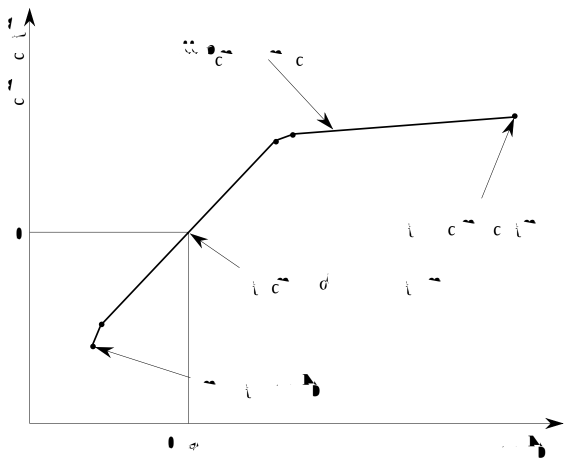
x f [illegible]:[illegible] x f [illegible] x_3 f [illegible]:333

r f [illegible] 3 r f [illegible] r_3 f [illegible] 3 r f [illegible]

d f [illegible] d f [illegible] d_3 f [illegible] d f [illegible]

ADR f [illegible] r_P f [illegible]

[illegible]



Incorrect Mean-ADR Formulation

[illegible] ADR

[illegible]

r f [illegible] x , [illegible] x , [illegible] x_3

r f [illegible] x , [illegible] x , [illegible] x_3

r_3 f [illegible] x , 3:[illegible] x [illegible] x_3

r f [illegible] x :3 [illegible] x , [illegible]:3 [illegible] x_3

r_P f [illegible] r , [illegible] r , [illegible] r_3 , [illegible] r

[illegible] r_P r_P

d f [illegible] r > [illegible] r [illegible]

d f [illegible] r > [illegible] r [illegible]

d_3 f [illegible] r_3 > [illegible] r_3 [illegible]

d f [illegible] r > [illegible] r [illegible]

ADR f [illegible] d , [illegible] d , [illegible] d_3 , [illegible] d

x , x , x_3 f

x_j [illegible]; j f ; ;3

[illegible] a a [illegible] d_i

d_i f [illegible] r_i > [illegible]; [illegible]; r_i[illegible];

[illegible] ff [illegible] a [illegible] r_i f [illegible]

[illegible]

Mean-ADR and Mean-Variance Analysis Compared

Mean-Variance Optimization Model

Mean-Variance Spreadsheet Model

=STDEVP(D13:D16)

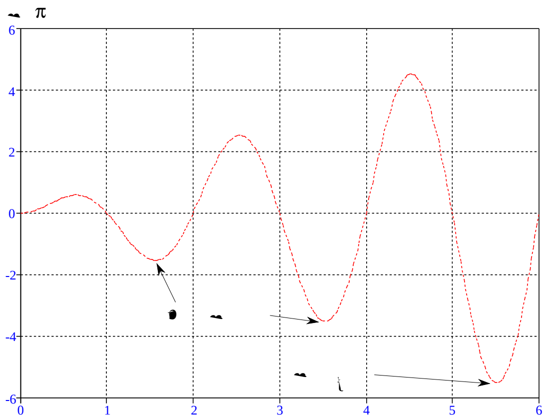
=AVERAGE(D13:D16)

A	A	B	C	D	E	F	G	H
1	INV_MV.XLS		Mean-Variance Optimization Model					
2								
3						sum of	budget	
4	Standard			Avg. return		port wts	constraint	
5	Deviation			r(P)		1.000	=	1
6	1.791			1.400				
7				>=		portfolio weights x(j)		
8			Min return:	1.4		0.000	0.667	0.333
9								
10				Return		Scenario returns r(i,j)		
11				by scen		Securities		
12				r(i)	Scenario	1	2	3
13				4.053	1	5.51	4.80	2.56
14				0.460	2	-1.24	0.61	0.16
15				1.853	3	5.46	3.60	-1.64
16				-0.767	4	-1.70	-1.30	0.30

=SUMPRODUCT($F$8:$H$8,F13:H13)

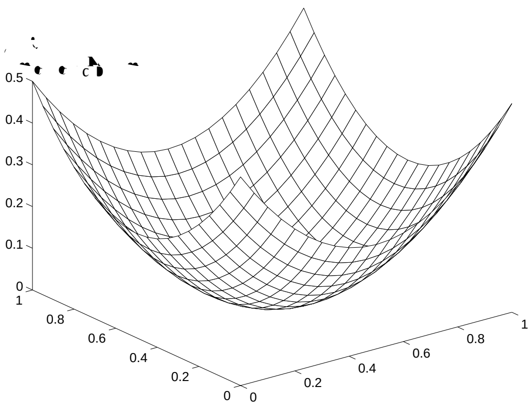
Comparison of Approaches

Nonlinear Programming



Nonlinear Programming (continued)

Nonlinear Programming (continued)



Summary

For next class