COLUMBIA BUSINESS SCHOOL

Decision Models

Lecture 3

Bidding Problems

Multiperiod Planning Models

Summary and Preparation for next class

The Bidding Problem

Petromor is selling land with good oil extraction potential.
Oil companies present sealed offers ($ per barrel) for the zones that they are interested in buying.
No oil company can be awarded more than one zone as a result of the public offering.
Petromor would like to maximize the revenue resulting from these sales.

Table 1. Bids (in $ per Barrel)

	A	B	C	D	E	F
Zone 1	$8.75	$8.70	$8.80	$8.65	$8.60	$8.50
Zone 2	$6.80	$7.15	$7.25	$7.00	$7.20	$6.85
Zone 3	$8.30	$8.20	$8.70	$7.90	$8.50	$8.40
Zone 4	$7.60	$8.00	$8.10	$8.00	$8.05	$7.85

Table 2. Zone potential (in # of barrels)

	Potential
Zone 1	205,000
Zone 2	240,000
Zone 3	215,000
Zone 4	225,000

What is the most profitable assignment of zones to the companies in this case?

Petromor Bidding Formulation

Indices: To index the zones, let ί ƒ 1, 2, 3, 4. To index the companies, let ƒ A, B, . . ., F.

Decision Variables: Let

ί ƒ { 1 if zone ί is assigned to company ,
0 otherwise

Objective Function:

Maximize total sales revenue:

max 205,000(8.75 $_{1A}$, 8.70 $_{1B}$, , 8.50 $_{1F}$
, 240,000(6.80 $_{2A}$, 7.20 $_{2B}$, , 6.85 $_{2F}$
, 215,000(8.30 $_{3A}$, 8.20 $_{3B}$, , 8.40 $_{3F}$
, 225,000(7.60 $_{4A}$, 8.00 $_{4B}$, , 7.85 $_{4F}$

Constraints:

Every zone must be assigned to some company

Total number of companies assigned to each zone ƒ 1

This leads to four constraints:

(Zone 1) $_{1A}$, $_{1B}$, $_{1C}$, $_{1D}$, $_{1E}$, $_{1F}$ ƒ 1
(Zone 2) $_{2A}$, $_{2B}$, $_{2C}$, $_{2D}$, $_{2E}$, $_{2F}$ ƒ 1
(Zone 3) $_{3A}$, $_{3B}$, $_{3C}$, $_{3D}$, $_{3E}$, $_{3F}$ ƒ 1
(Zone 4) $_{4A}$, $_{4B}$, $_{4C}$, $_{4D}$, $_{4E}$, $_{4F}$ ƒ 1

Petromor Bidding Formulation (continued)

Constraints (continued):

Every company can be assigned at most one zone

Total number of zones assigned to each company 1

This leads to six constraints:

(Company A) $_{1A}$, $_{2A}$, $_{3A}$, $_{4A}$ 1
(Company B) $_{1B}$, $_{2B}$, $_{3B}$, $_{4B}$ 1
(Company C) $_{1C}$, $_{2C}$, $_{3C}$, $_{4C}$ 1
(Company D) $_{1D}$, $_{2D}$, $_{3D}$, $_{4D}$ 1
(Company E) $_{1E}$, $_{2E}$, $_{3E}$, $_{4E}$ 1
(Company F) $_{1F}$, $_{2F}$, $_{3F}$, $_{4F}$ 1

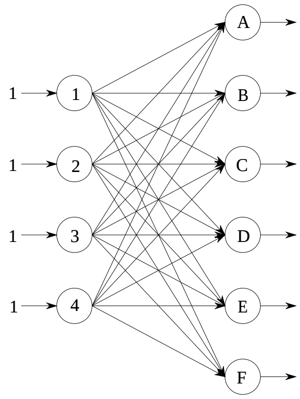
Finally, the nonnegativity constraints:

ί 0, ί ƒ 1, 2, 3, 4, ƒ A, B, C, D, E, F.

Should we add constraints restricting the decision variables to take on integer values only?

Network Model

It is *not* necessary to restrict the decision variables to take integer values. This will occur automatically, since the formulation is a network linear program.



Constraints:

For every zone:

Total bids out ƒ 1

For every company:

Total bids in 1

Assignment Models

Since there are no transshipment nodes, and since the supply at each source is one, the model is called an *assignment model.* These models are frequently used for:

Assigning tasks to workers/machines

! For scheduling operations

! Classrooms, roommate assignments

Bidding for Awards and Contracts:

! The New York City Department of Sanitation uses a similar model to assign contracts for garbage disposal.

! The Bureau of Land Management of the Ministry of the Interior holds bimonthly simultaneous drawings enabling the public to acquire leases on large land parcels. A multibillion dollar industry of professional filing services assists investors in selecting parcels. One of these firms uses a similar model to assign clients to land parcel applications.

Bidding Problem Optimized Spreadsheet

Objective function
=SUMPRODUCT(B20:G23,B13:G16)

	A	B	C	D	E	F	G	H	I	J
1	PETROMOR.XLS			**Petromor Oil Company**						
2										
3				**Revenue (in '000)**			$7,192.3			
4	**Bids (in $ per barrel)**								**Extraction**	
5		A	B	C	D	E	F		**Potential**	
6	Zone 1	$8.75	$8.70	$8.80	$8.65	$8.60	$8.50		205,000	
7	Zone 2	$6.80	$7.15	$7.25	$7.00	$7.20	$6.85		240,000	
8	Zone 3	$8.30	$8.20	$8.70	$7.90	$8.50	$8.40		215,000	
9	Zone 4	$7.60	$8.00	$8.10	$8.00	$8.05	$7.85		225,000	
10										
11	**Bids per well (in thousands)**									
12		A	B	C	D	E	F			
13	Zone 1	$1,794	$1,784	$1,804	$1,773	$1,763	$1,743			
14	Zone 2	$1,632	$1,716	$1,740	$1,680	$1,728	$1,644			
15	Zone 3	$1,785	$1,763	$1,871	$1,699	$1,828	$1,806			
16	Zone 4	$1,710	$1,800	$1,823	$1,800	$1,811	$1,766			
17										
18	**Bids Assigned**									
19		A	B	C	D	E	F	Total		
20	Zone 1	1	0	0	0	0	0	1		
21	Zone 2	0	0	0	0	1	0	1		
22	Zone 3	0	0	1	0	0	0	1		
23	Zone 4	0	1	0	0	0	0	1		
24	Total	1	1	1	0	1	0			

H20:H23 constrained = 1

Decision variables B20:G23, constrained between 0 and 1

B24:G24 constrained <= 1

Decision variables: Located in cells B20:G23.

Objective function to be maximized is cell G3.

Constraints are indicated in the spreadsheet.

Bidding Problem Sensitivity Report

Changing Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B20	Zone 1 A	1	0	1793.75	1E+30	10.25000006
C20	Zone 1 B	0	0	1783.5	10.25000006	2.749999907
D20	Zone 1 C	0	-3	1804	2.749999907	1E+30
E20	Zone 1 D	0	-10	1773.25	10.25	1E+30
F20	Zone 1 E	0	-32	1763	31.74999991	1E+30
G20	Zone 1 F	0	-41	1742.5	40.99999995	1E+30
B21	Zone 2 A	0	-95	1632	95.00000011	1E+30
C21	Zone 2 B	0	-1	1716	0.750000005	1E+30
D21	Zone 2 C	0	0	1740	31.00000001	0.74999997
E21	Zone 2 D	0	-37	1680	36.74999999	1E+30
F21	Zone 2 E	1	0	1728	0.74999997	0.750000005
G21	Zone 2 F	0	-73	1644	72.75000008	1E+30
B22	Zone 3 A	0	-73	1784.5	73.00000018	1E+30
C22	Zone 3 B	0	-84	1763	84.25000014	1E+30
D22	Zone 3 C	1	0	1870.5	1E+30	31.00000001
E22	Zone 3 D	0	-149	1698.5	148.75	1E+30
F22	Zone 3 E	0	-31	1827.5	31.00000001	1E+30
G22	Zone 3 F	0	-41	1806	41.25000003	1E+30
B23	Zone 4 A	0	-100	1710	100.2500001	1E+30
C23	Zone 4 B	1	0	1800	2.749999907	0
D23	Zone 4 C	0	-1	1822.5	0.74999997	1E+30
E23	Zone 4 D	0	0	1800	0	10.25
F23	Zone 4 E	0	0	1811.25	0.750000005	0.74999997
G23	Zone 4 F	0	-34	1766.25	33.74999997	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
H20	Zone 1 Total	1	1,783	1	1	0
H21	Zone 2 Total	1	1,717	1	0	0
H22	Zone 3 Total	1	1,847	1	0	0
H23	Zone 4 Total	1	1,800	1	1	0
B24	Total A	1	10	1	0	1
C24	Total B	1	0	1	0	1
D24	Total C	1	23	1	0	0
E24	Total D	0	0	1	1E+30	1
F24	Total E	1	11	1	0	0
G24	Total F	0	0	1	1E+30	1

Petromor Bidding Optimal Solution

	Zone 1	Zone 2	Zone 3	Zone 4
Company Assigned:	A	E	C	B

Total revenue from the sales: $7,192.3 thousand.

Dual prices and RHS ranges for flow balance constraints (for each bidder):

Company	Dual Price	Allowable Increase	Allowable Decrease
A	10.25	0	1
B	0	0	1
C	23.25	0	0
D	0	Infinity	1
E	11.25	0	0
F	0	Infinity	1

Extra decimal places in the dual prices are obtained by changing the numeric format of the Excel sensitivity report.

Interpretation of the Sensitivity Report I

Company D is a fake company created by the owners of Company A, so as to circumvent the restriction that no more than one zone can be assigned to a company. Company D should have been eliminated from the bid.

Would the result of the optimization have been different?

No, because Company D was not assigned any zones. This means that the dual price associated with the constraint limiting the number of bids assigned to Company D is zero, and hence, any changes in the RHS will not affect the optimal solution.

Interpretation of the Sensitivity Report II

After the envelopes with all the bids have been opened, all the bidding companies can find out what the other companies offered for the different zones. Mr. Vaco overheard the following statement from a senior analyst at company A: “Our offer was too high; we could have lowered it by almost $0.10 a barrel, and still have been awarded Zone 1.”

Is it true that Company A could have lowered their bid for Zone 1 by $0.10 and still have won the bidding?

From the sensitivity report, we can see that the objective function coefficient for Zone 1, Company A, could have been decreased by $10,250 without affecting the result of the optimization. This means that Company A could have decreased their bid by at most $0.05 per barrel (=$10,250/205,000) and still have won the bid. A decrease of $0.10 per barrel is outside the range, so we would have to reoptimize to get the correct solution. This new solution does not assign Zone 1 to Company A.

Interpretation of the Sensitivity Report III

What would happen if Company A decided to pull out from the bid?

We can answer this question by looking at the dual price associated with Company A. If we do not assign any zones to Company A then the revenue would go down by $10,250 (the RHS goes from 1 to 0, and the decrease is within the allowable decrease of 1).

What is the “hidden cost” of the policy that each company can be assigned at most one zone?

If each company can be assigned any number of zones, we need to delete the six company constraints “Total bids in 1” (i.e., the constraints on cells B24:G24 should be deleted). Since this question involves a change to six constraints, we need to reoptimize the model.

The optimal revenue increases by $44,750 to $7,237,000. That is, the hidden cost of the policy that each company can be assigned to at most one zone is $44,750.

Multiperiod Planning Models

In many settings we need to plan over a time horizon of many periods because

decisions for the current planning period affect the future

requirements in the future need action now

Examples include:

Production / inventory planning

Human resource staffing

Investment problems

Capacity expansion / plant location problems

National Steel Corporation

National Steel Corporation (NSC) produces a special-purpose steel used in the aircraft and aerospace industries. The sales department has received orders for the next four months:

	Jan	Feb	Mar	Apr
Demand (tons)	2300	2000	3100	3000

NSC can meet demand by producing the steel, by drawing from its inventory, or a combination of these. Inventory at the beginning of January is zero. Production costs are expected to rise in Feb and Mar. Production and inventory costs are:

	Jan	Feb	Mar	Apr
Production cost	3000	3300	3600	3600
Inventory cost	250	250	250	250

Production costs are in $ per ton. Inventory costs are in $ per ton per month. For example, 1 ton in inventory for 1 month costs $250; for 2 months, it costs $500.

NSC can produce at most 3000 tons of steel per month. What production plan meets demand at minimum cost?

NSC Production Model Overview

What needs to be decided?

A production plan, i.e., the amount of steel to produce in each of the next 4 months.

What is the objective?

Minimize the total production and inventory cost. These costs must be calculated from the decision variables.

What are the constraints?

Demand must be met each month. Constraints to define inventory in each month. (We'll see later that these constraints can be done together as "flow balance" constraints.) Production capacity constraints. Nonnegativity.

NSC optimization model in general terms:

min Total Production plus Inventory Cost

subject to:

Production capacity constraints

Flow balance constraints

Nonnegative production and inventory

NSC Multiperiod Production Model

Index: Let $t = 1, 2, 3, 4$ represent the months Jan, Feb, Mar, and Apr, respectively.

Decision Variables: Let

$P_t =$ # of tons of steel to produce in month t

$I_t =$ # of tons of inventory from month t to $t + 1$

Note: The production variables P_t are the *main* decision variables, because the inventory levels are determined once the production levels are set. Often the P_t's are called *controllable* decision variables and the I_t's are called *uncontrollable* decision variables.

Objective Function:

The total cost is the sum of production and inventory cost. Total production cost, *PROD*, is:

$$PROD = 3000P_1 + 3300P_2 + 3600P_3 + 3600P_4.$$

Total inventory cost, *INV*, is:

$$INV = 250I_1 + 250I_2 + 250I_3 + 250I_4.$$

Demand Constraints

In order to meet demand in the first month, we want

$$P_1 \geq 2300.$$

Set

$$I_1 = P_1 - 2300$$

and note that $P_1 \geq 2300$ is equivalent to $I_1 \geq 0$.

In order to meet demand in the second month, the tons of steel available must be at least 2000:

$$I_1 + P_2 \geq 2000.$$

Set

$$I_2 = I_1 + P_2 - 2000$$

and note that $I_1 + P_2 \geq 2000$ is equivalent to $I_2 \geq 0$.

The inventory and nonnegativity constraints:

(Month 1) $I_1 = P_1 - 2300, \quad I_1 \geq 0$

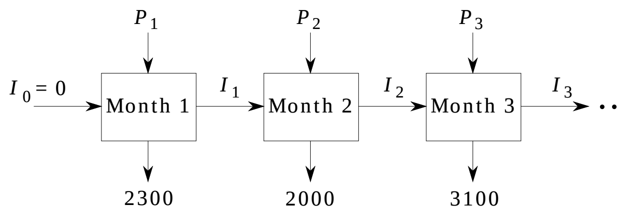
(Month 2) $I_2 = I_1 + P_2 - 2000, \quad I_2 \geq 0$

(Month 3) $I_3 = I_2 + P_3 - 3100, \quad I_3 \geq 0$

define the inventory decision variables and enforce the demand constraints.

NSC Production Model (continued)

Another way to view the constraints: The inventory variables *link* one period to the next. The inventory definition constraints can be visualized as "flow balance" constraints:



Flow balance constraints for each month:

"Flow in = Flow out"

(Month 1) $P_1 = I_1 + 2300$

(Month 2) $I_1 + P_2 = I_2 + 2000$

(Month 3) $I_2 + P_3 = I_3 + 3100$

$\vdots$

Are there any other constraints? Production cannot exceed 3000 tons in any month:

$$P_i \leq 3000 \quad \text{for } i = 1, 2, 3, 4.$$

NSC Linear Programming Model

$$\min \; PROD + INV$$

subject to:

Cost definitions:

(*PROD* Def.) $PROD = 3000P_1 + 3300P_2 + 3600P_3 + 3600P_4$

(*INV* Def.) $INV = 250I_1 + 250I_2 + 250I_3 + 250I_4$

Production capacity constraints:

$P_i \quad 3000 \quad i = 1, 2, 3, 4$

Inventory balance constraints:

"Flow in = Flow out"

(Month 1) $P_1 = I_1 + 2300$

(Month 2) $I_1 + P_2 = I_2 + 2000$

(Month 3) $I_2 + P_3 = I_3 + 3100$

(Month 4) $I_3 + P_4 = I_4 + 3000$

Nonnegativity: All variables 0

NSC Optimized Spreadsheet

=SUMPRODUCT(D8:G8,D13:G13)/1000

=SUMPRODUCT(D9:G9,D15:G15)/1000

Objective Function
=+D3+D4

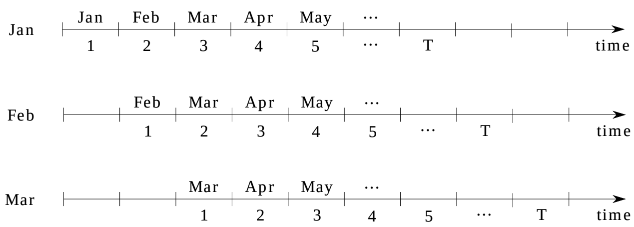
A	A	B	C	D	E	F	G	H
1	NSC.XLS		National Steel Corporation					
2								
3	Production cost (in $1000)			34,740				
4	Inventory cost (in $1000)			600				
5	Total cost (in $1000)			$35,340				
6								
7	Unit costs:			Jan	Feb	Mar	Apr	
8	Variable production cost ($/ton)			3000	3300	3600	3600	
9	Inventory cost ($/ton per month)			250	250	250	250	
10								
11				Jan	Feb	Mar	Apr	
12	Beginning Inventory..................			0	700	1700	0	
13	Production Level.......................			3000	3000	1400	3000	
14	Demand....................................			2300	2000	3100	3000	
15	Ending Inventory.......................			700	1700	0	0	
16								
17	Inventory >= 0 Constraints.........			>= 0	>= 0	>= 0	>= 0	
18	Production <= 3000 Constraints			<=	<=	<=	<=	
19								

=+D12+D13-D14

The optimal solution has a total cost of $35,340,000.

Multiperiod Models in Practice

Most multiperiod planning systems operate on a *rolling horizon basis:*



A T-period model is solved in January and the optimal solution is used to determine the plan for January. In February, a new T-period model is solved, incorporating updated forecasts and other new information. The optimal solution is used to determine the plan for February.

Often long horizon models are used to estimate needed capacity and determine aggregate planning decisions (*strategic issues*). Then more detailed short horizon models are used to determine daily and weekly operating decisions (*tactical issues*).

Summary

Petromor Assignment Model

! Understanding the sensitivity report

Multiperiod Planning Models

! Inventory balance constraints

! Linking periods together with constraints

For next class

Read Chapter 3.7 in the W&A text.
Read and think about the “Foreign Currency Trading” case, p.146 in the W&A text. (You are not expected to solve this case before the next class.)